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SUBJECT NAME: ADVANCED DISCRETE MATHEMATICS

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Advanced

Discrete mathematics

Logic. logic is a rule by which one can predict whether a given argument or reasoning is valid or not. In computer science, it is used to verify the correctness of the programs.

Statement: A statement (or proposition), is a sentence which is either true or false

ex ① $8 > 10$

It is a statement as its truth value is F. (False)

② Blood is Read its truth value is T (true)

Note. If a sentence is a question then it is not statement.

Negation of the statement. symbol

Let p: Delhi is a city

$\sim p$: Delhi is not a city [it is negation of p]

Truth value table

Let p and q be any two statements then $(p \wedge q)$ is called conjuction of p and q.

its truth table is given by

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

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Disjunction ($p \vee q$) [Here $\vee$ is called connective used to form combined statement.)

Truth table for Disjuction

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Conditional connective " $\rightarrow$ "

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Bi- conditional connetive " $\Longleftrightarrow$ "

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

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Converse of $p \to q$ is $q \to p$

p	q	$p \to q$	conv$(p \to q)$ $= q \to p$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

Inverse of $p \to q$ = $\sim p \to \sim q$

p	q	$\sim p$	$\sim q$	$p \to q$	$\sim p \to \sim q$
T	T	F	F	T	T
T	F	F	T	F	T
F	T	T	F	T	F
F	F	T	T	T	T

Contrapositive of $p \to q$ = $\sim q \to \sim p$

p	q	$\sim q$	$\sim p$	$p \to q$	$\sim q \to \sim p$
T	T	F	F	T	T
T	F	T	F	F	F
F	T	F	T	T	T
F	F	T	T	T	T

Tautologies: A statement (or propositional function) which is true for all possible truth values is called a tautology.

Example: ① Show $[p \wedge (p \to q)] \to q$ is a tautology

p	q	$p \to q$	$p \wedge (p \to q)$	$p \wedge (p \to q) \to q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Proved.

Example: ② Show $[(p \to q) \wedge (q \to r)] \to (p \to r)$ is a tautology.

p	q	r	$p \to q$	$q \to r$	$a = (p \to q) \wedge (q \to r)$	$p \to r$	$[(p \to q) \wedge (q \to r)] \to (p \to r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	T	T
T	F	T	F	T	F	F	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

which is a tautology

contradiction. A statement which is always false is called a contradiction

ex

p	$\sim p$	$p \wedge \sim p$
T	F	F
F	T	F

Equivalent statements. Two statements are said to be equivalent if their truth values are same.

e.g Prove $p \to q = \sim p \vee q$

Ans

p	q	$\sim p$	$\sim p \vee q$	$p \to q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

Some important Laws

(1) Commutative law (a) $p \vee q = q \vee p$ (b) $p \wedge q = q \wedge p$

(2) Associative law (a) $(p \vee q) \vee r = p \vee (q \vee r)$
(b) $(p \wedge q) \wedge r = p \wedge (q \wedge r)$

(3) Distributive law (a) $p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$
(b) $p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$

(4) Idempotent law. (a) $p \vee p = p$ (b) $p \wedge p = p$

(5) Law of absorption. (a) $p \vee (p \wedge q) = p$ (b) $p \wedge (p \vee q) = p$

(6) Involution laws. $\sim(\sim p) = p$

(7) Complement law. (a) $p \vee \sim p = T$ (b) $p \wedge \sim p = F$

(8) Demorgan's law (a) $\sim(p \vee q) = \sim p \wedge \sim q$
(b) $\sim(p \wedge q) = \sim p \vee \sim q$

(9) operation with T. (a) $p \vee T = T$ (b) $p \wedge T = p$
operation with F (a) $p \vee F = p$ (b) $p \wedge F = F$

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Remember.

(1) $p \to q = \sim p \vee q$

(2) $p \leftrightarrow q = (p \to q) \wedge (q \to p)$

(3) $\sim (p \to q) = p \wedge \sim q$

(4) $\sim (p \leftrightarrow q) = p \leftrightarrow \sim q$

Imp Question. Without using truth table prove

$\sim (p \wedge q) \to (\sim p \vee (\sim p \vee q)) = \sim p \vee q$

$LHS = \sim (p \wedge q) \to (\sim p \vee (\sim p \vee q))$

$LHS = (p \wedge q) \vee \{\sim p \vee (\sim p \vee q)\}$ $\quad p \to q = \sim p \vee q$

$LHS = (p \wedge q) \vee \{(\sim p \vee \sim p) \vee q\}$ $\quad$ Associativity

$LHS = (p \wedge q) \vee \{\sim p \vee q\}$ $\quad [\sim p \vee \sim p = \sim p$ Idempotent law$]$

$LHS = \{(p \wedge q) \vee (\sim p)\} \vee q$ $\quad$ (Associativity]

$LHS = \{\sim p \vee (p \wedge q)\} \vee q$ $\quad$ [commutative]

$LHS = \{(\sim p \vee p) \wedge (\sim p \vee q)\} \vee q$

$LHS = \{T \wedge (\sim p \vee q)\} \vee q$

$LHS = (\sim p \vee q) \vee q$ $\quad [T \wedge p = p]$

$LHS = \sim p \vee (q \vee q)$ $\quad$ (Associativity]

$LHS = \sim p \vee q$ $\quad [\because q \vee q = q]$

$LHS = RHS.$

Functionally complete set. A set of operations (or connectives) is called a functionally complete if any statement (or compound) can be expressed entirely in terms of given operations equivalently.

ex The set $\{\rightarrow, \sim\}$ is functionally complete as $(p \vee q)$ is any statement This can be expressed also as

$$p \vee q = \sim p \rightarrow q$$

i.e interms of given operations equivalently

(2) $p \rightarrow q = \sim p \vee q$

$\Rightarrow$ $(\sim, \vee)$ is functionally complete

(3) $\sim(p \rightarrow q) = p \wedge \sim q$

$\Rightarrow$ $(\wedge, \sim)$ is functionally complete

(4) $(\rightarrow)$ is functionally complete.

(5) $(\vee, \wedge)$ is not functionally complete as There is not statement when can be expressed Using a $(\vee, \wedge)$ equivalently.

<u>Some special connectives</u>. $(\uparrow)$, $(\downarrow)$

(a) $p \uparrow q = \sim(p \wedge q)$

symbol $\rightarrow$ $\uparrow$ $\rightarrow$ NAND

(b) $p \downarrow q = \sim(p \vee q)$

symbol: $\downarrow$ = NOR

p	q	$p\wedge q$	$\sim(p\wedge q)$	$p\uparrow q$	$p\vee q$	$\sim(p\vee q)$	$p\downarrow q$
T	T	T	F	F	T	F	F
T	F	F	T	T	T	F	F
F	T	F	T	T	T	F	F
F	F	F	T	T	F	T	T
			✓	✓		✓✓	✓✓

Clearly $\left.\begin{array}{l}\sim(p\wedge q) = p\uparrow q \\ \sim(p\vee q) = p\downarrow q\end{array}\right\}.$

Quantifiers

The words some, no and all are known as quantifiers

Types of quantifier.

① Universal quantifier ② Existential quantifier

① Universal quantifier. The symbol $\forall$ for all (or for any or for every) is called universal quantifier

ex Let $p: n+4 > 3, \forall n \in N$

② Existantial quantifier:
The symbol $\exists$ (there exists), for atleast one or "for some" is called existancial quantifier

ex $\exists x \mid x^2 - 4x = 16$

③ Negation, $\sim[\exists_x p] = \forall_x (\sim p)$

$\sim[\forall_x p] = \exists_x (\sim p)$

Valid argument. An argument is said to be valid iff the conjunction of the premises implies conclusion

ex

$$\begin{array}{c} p_1 \\ p_2 \\ p_3 \\ \vdots \\ p_n \\ \hline q \end{array}$$

i.e $(p_1 \wedge p_2 \wedge \dots \wedge p_n) \rightarrow q$ is a tautology.

Q Prove the following argument is valid

$$\begin{array}{c} p \\ p \rightarrow q \\ \hline q \end{array}$$

p	q	$p \rightarrow q$	$p \wedge (p \rightarrow q)$	$\{p \wedge (p \rightarrow q)\} \rightarrow q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	

Hence, given argument is valid as $\{p \wedge (p \rightarrow q)\} \rightarrow q$ is a tautology.

(a) **Modus Ponens**

$$\begin{array}{c} p \\ p \rightarrow q \\ \hline q \end{array}$$

this argument which is valid is called modus Ponens.

(b) Law of syllowgism. The following valid argument

$$\begin{array}{c} p \rightarrow q \\ q \rightarrow r \\ \hline p \rightarrow r \end{array}$$

is called law of syllogism.

Q. Given the following

(a) If he takes coffee, he does not drink milk

(b) He eats crackers only if he drink milk

(c) He does not take soup unless he eats crackers

(d) At noon today, he had coffee

Whether he took soup at noon today? If so what is the correct conclusion

Solⁿ. let p: he takes coffee

q: he drink milk

r: he eats crackes

s: he takes soup

then as per problem, we have

$$p \to \sim q \quad \text{——— (1)}$$
$$r \to q \quad \text{——— (2)}$$
$$\sim r \to \sim s \quad \text{——— (3)}$$
$$p \quad \text{——— (4)}$$
$$\overline{\quad ? \quad}$$

(1) ⟹ $p \to \sim q$

(2) ⟹ $\sim q \to \sim r$ (contrapositive of (2))

$$\overline{p \to \sim r} \quad \text{——— (5)}$$
$$\sim r \to \sim s$$
$$\overline{p \to \sim s} \quad \text{——— (6)}$$
$$p$$
$$\overline{\sim s} \quad \text{conclusion}$$

Conclusion: he did not take soup at noon today,