UNIT - 3

* Motion under the Central force:-

That force which Acts towards the fixed point And depend upon the magnitude of a radial distance is known as Central force.

Central force may be Attractive as well as repulsive in Nature It means that every force in which force of Attraction And Repulsion act is known as Central force

If the force is less then zero then it is Attractive in Nature If The force is greater Then zero it is repulsive in Nature

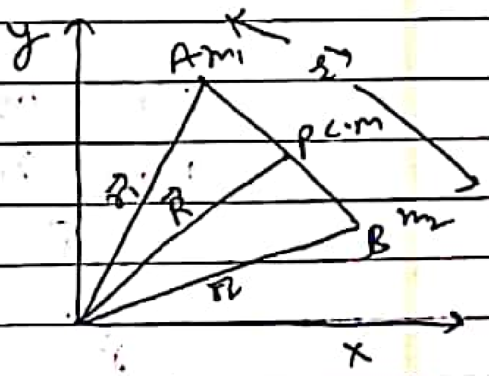
Central force may be written as

$$\vec{F} = \hat{r} F(r)$$

$\hat{r}$ = Unit Vector

$F(r)$ = magnitude of force

* Concepts of Reduced mass or Equalient two body problem or Reduced two body problem into one body problem:-



Let us Consider two points A & B at which two masses m_1 & m_2 are placed The distance of These masses from a fixed point is $\vec{r}_1$ And $\vec{r}_2$ respectively.

Let Center of mass of These body lie's at a point P which is at the middle of

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These two body Now the K.E. of the System can be written as

$$T = K.E.(\text{due of mass } m_1 m_2) + K.E(\text{due to } m_1) + K.E(\text{due}$$

$$T = \frac{1}{2}(m_1+m_2)\dot{\vec{R}}^2 + \frac{1}{2}m_1\dot{r}_1'^2 + \frac{1}{2}m_2\dot{r}_2'^2$$

Now we know that lagrange of Any system is Equal to the difference of K.E. And P.E.

$$L = T - V$$

$$L = \frac{1}{2}(m_1+m_2)\dot{\vec{R}}^2 + \frac{1}{2}m_1\dot{r}_1^2 + \frac{1}{2}m_2\dot{r}_2^2 - V(r) \quad (1)$$

Now we also know that

$$\vec{r} = \vec{r}_1' - \vec{r}_2' \quad (2)$$

Now in condition in balance.

$$m_1\vec{r}_1' + m_2\vec{r}_2' = 0$$

$$m_2\vec{r}_2 = -m_1\vec{r}_1$$

$$\vec{r}_2' = -\frac{m_1\vec{r}_1}{m_2}$$ put in Eq (2) we get

$$\vec{r} = \vec{r}_1' + \frac{m_1\vec{r}_1'}{m_2}$$

$$\vec{r} = \frac{m_2\vec{r}_1' + m_1\vec{r}_1'}{m_2}$$

$$m_2\vec{r} = m_2\vec{r}_1' + m_1\vec{r}_1'$$

$$m_2\vec{r} = \vec{r}_1'[m_2+m_1]$$

$$\vec{r}_1' = \frac{m_2\vec{r}}{m_1+m_2}$$

$$\dot{\vec{r}}_1' = \frac{m_2\dot{\vec{r}}}{m_1+m_2} \quad (3)$$

Similarly $\dot{\vec{r}}_2' = \frac{m_1\dot{\vec{r}}}{m_1+m_2}$

Putting the value of $\vec{r}_1$ And $\vec{r}_2$ in Eq (1) we get

$$L = \frac{1}{2}(m_1+m_2)\dot{\vec{R}}^2 + \frac{1}{2}m_1\left[\frac{m_2\dot{\vec{r}}}{m_1+m_2}\right]^2 + \frac{1}{2}m_2\left[\frac{-m_1\dot{\vec{r}}}{m_1+m_2}\right]^2 - V(r)$$

$$= \frac{1}{2}(m_1+m_2)\dot{\vec{R}}^2 + \frac{1}{2}\frac{m_1 m_2^2\dot{r}^2}{(m_1+m_2)^2} + \frac{1}{2}m_2\frac{m_1^2\dot{r}^2}{(m_1+m_2)^2} - V(r)$$

$$= \frac{1}{2}(m_1+m_2)\dot{R}^2 + \frac{1}{2}\frac{m_1 m_2}{m_1+m_2}\dot{r}^2 - V(r)$$

Now Let us define

$$m_1+m_2 = M$$

$$\frac{m_1 m_2}{m_1+m_2} = \mu$$

$$\frac{1}{2}M\dot{R}^2 + \frac{1}{2}\mu\dot{r}^2 - V(r) \quad \text{--- (4)}$$

Now There are two Variable R & μ in above Equation. The Co-ordinate one is a cyclic because it does not present in the above system i.e. $\frac{\partial L}{\partial R} = 0$ Then the Lagrange Equation of motion

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{R}}\right) - \frac{\partial L}{\partial R} = 0 \quad \text{--- (5)}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{r}}\right) - \frac{\partial L}{\partial r} = 0 \quad \text{--- (6)}$$

diff w.r.to $\frac{\partial L}{\partial \dot{R}}$, $\frac{\partial L}{\partial R}$, $\frac{\partial L}{\partial \dot{r}}$, $\frac{\partial L}{\partial r}$ of Eq (4) ~~we get~~

$$\frac{\partial L}{\partial \dot{R}} = M\dot{R}, \quad \frac{\partial L}{\partial R} = 0$$

$$\frac{\partial L}{\partial \dot{r}} = \mu\dot{r}, \quad \frac{\partial L}{\partial r} = -\frac{\partial V}{\partial r}$$

Putting these value in Eq (5) we get

$$\frac{d}{dt}(\mu \dot{R}) = 0 \Rightarrow \text{On Integrating}$$

$$\boxed{M\dot{R} = \text{Constant}}$$

From above it is clear that the total velocity due to Central force is always Conservative

From Eq (6).

$$\frac{d}{dt}(\mu \dot{r}) + \frac{dV}{dr} = 0$$

$$\mu \ddot{r} = -\frac{dV}{dr}$$

We know that -ve gradients of Potential Energy is known as Conservative force Then the Equation becomes:

$$\mu \ddot{r} = f(r)$$

$$\boxed{f(r) = \mu \ddot{r}}$$

This is the Equation of motion in the form of Reduced mass.

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