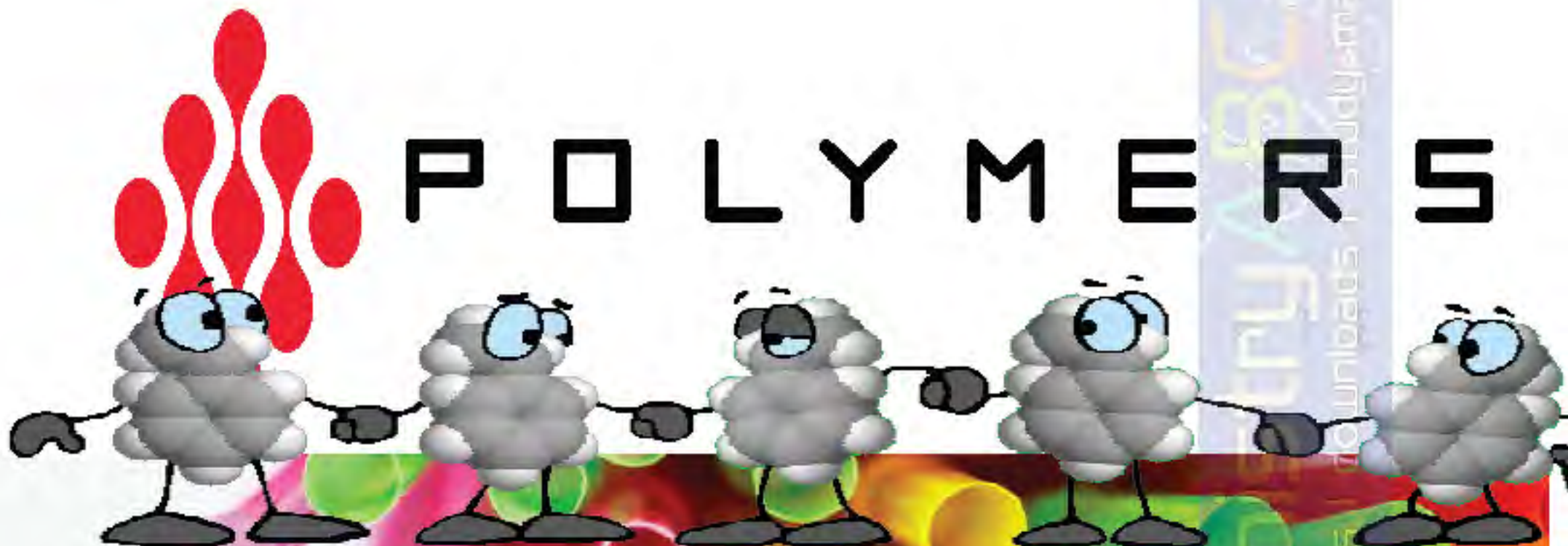


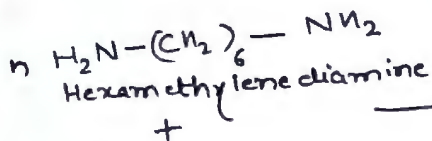
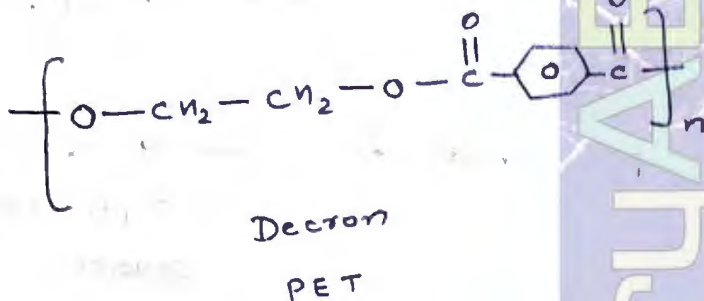
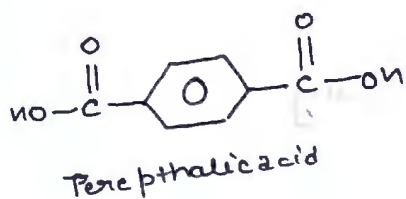
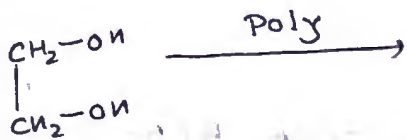
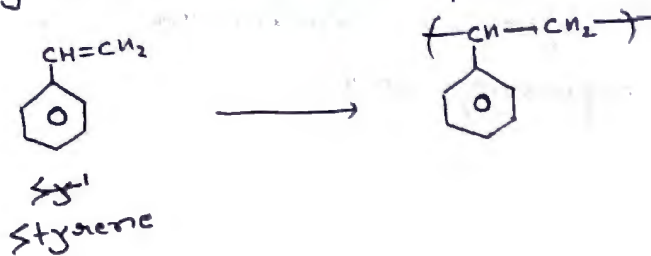
# POLYMERS



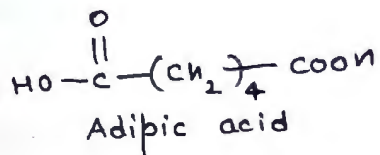
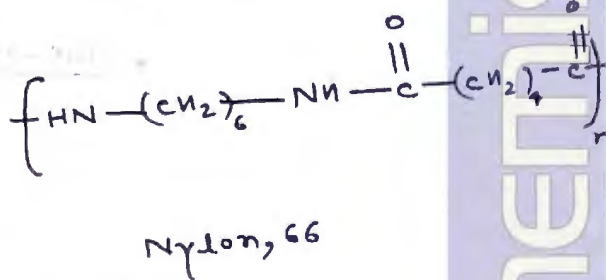
**BHAWANA (QUANTUM GIRL)**

# Polymers

- Made from combination of two monomers.



Polymerization

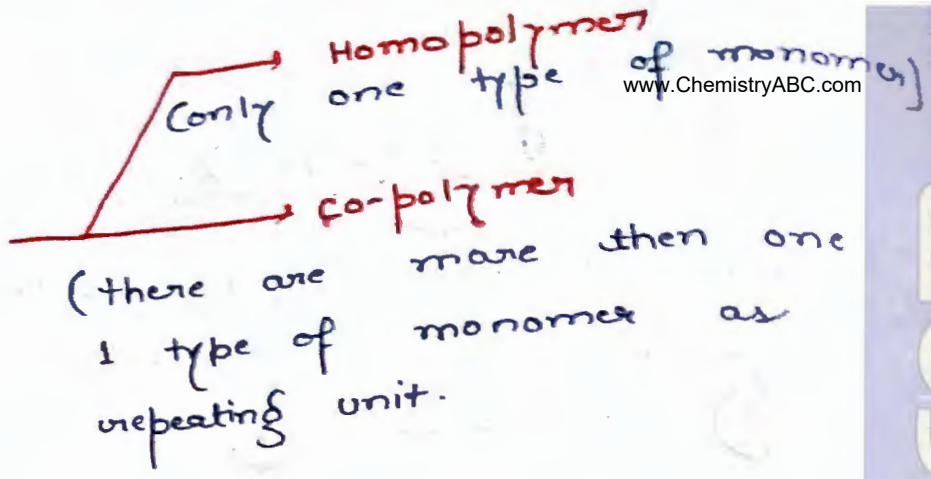


- Each polymer has a basic structure unit, that repeat it several times.
- Many of the polymer are prepared from only one type of monomer unit.  
e.g. styrene (poly)
- With more than one polymer  
e.g. PET, Nylon 66

2

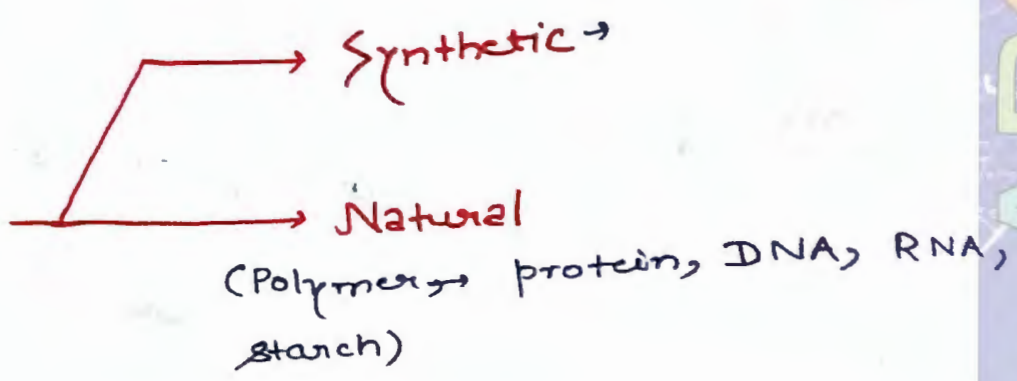
# Polymer

(In terms of repeating unit)

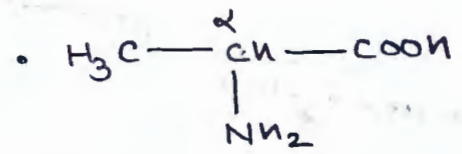


# Polymer

(



- Protein → naturally occurring polymers
- & Imp.
- Monomer - Amino acid



α-Alanine

# Polymerization →



The no of repeating unit, that is repeated again & again, is called degree of

polymerization

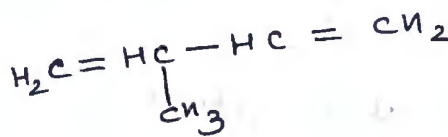
ChemistryABC.com

• a high molecular weight polymer is formed generally represent the no of monomeric unit contained in the polymer, it is designated by 'p'. if molar mass of polymer is 'M' & 'm' is the molar mass of monomeric unit

Then

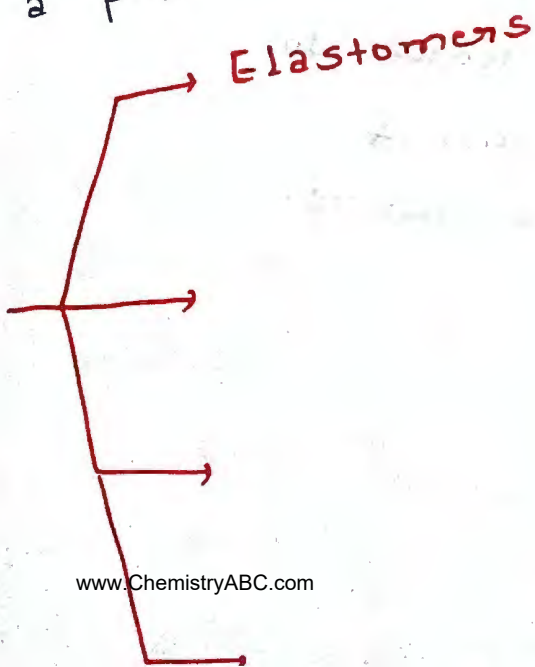
$$p = \frac{M}{m}$$

→ Natural rubber - is the polymer of isoprene (1,3-butadiene)



→ Latex - dispersed phase in aqueous medium coagulated by acid coagulation method.

→ Isoelectric point is weak reached to  
2 pI<sub>at</sub> 5.1 + 4.8



## Elastomers

- can be stretched
- weak intermolecular forces
- Regain its original shape,

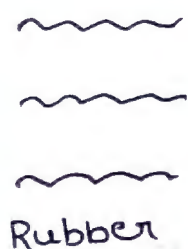
When stress is removed.

e.g. vulcanized rubber.

- Natural rubber - has no poor elasticity
- Soft & sticky when heated.

& become permanently deformed.

However, when natural rubber is heated with 3-5% sulphur, it becomes more elastic.



vulcanization



## Fibres →

- These have strong intermolecular forces.
- e.g. Polyamide (Nylon 66)  
Polyesters (PET/Dacron)
- These forces also leads to close packing of chains & thus give crystalline nature.
- Have sharp M.P.

Thermoplastics - Easily softened when heated & hardened when cooled. with a little change in their properties. The intermolecular forces are intermediate between Elastomers & fibres.

There is no cross linking b/w chains. Softening occurs when polymer chain move more & more freely because of the absence of cross-linking.

When heated form a fluid, which can be moulded into any desired shape & then cool cooled.

e.g. Polyethylene, Polystyrene, PVC, Teflon.

## Thermosetting polymers →

These are the polymers which undergo permanent change on heating, on heating they undergoes extensive cross linking & became hard & infusible

Therefore they cannot be reused.

e. They are generally prepared from low molecular mass semifluid substances.

They get highly cross linked to form hard, infusible & insoluble product.

cross-linking holds the molecule in place, so the heating doesn't allow them

e.g. Bakelite, Melamine formaldehyde resin.

## Plasticizers -

• PVC - made soft by di-n-butyl phthalates (plasticizers).

- Some other Plasticizers -
  - \* Dialkyl phthalates.
  - \* Cresyl phthalates.
  - \*



## Synthetic rubber -

Is any vulcanized rubber capable of getting stretched to about twice of its length.

on applying external force, it returns to its original shape & size as external stretching force is removed.

Most of these polymers are derived from butadiene derivatives containing carbon carbon double bond.

Synthetic rubber are either

(i) Homopolymer of 1,3-butadiene

(ii) copolymer of - one 1,3-butadiene derivative

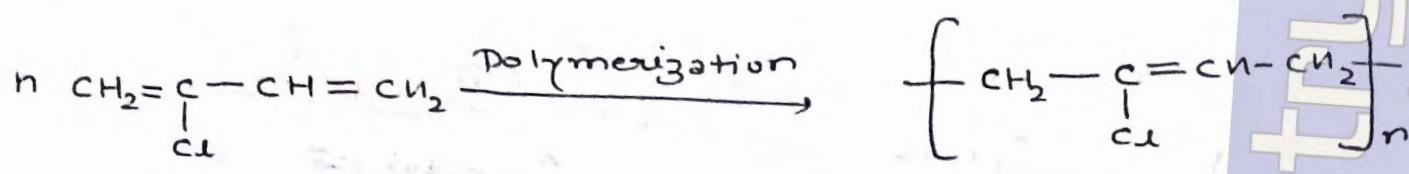
+ & other is

e.g. of Synthetic rubber

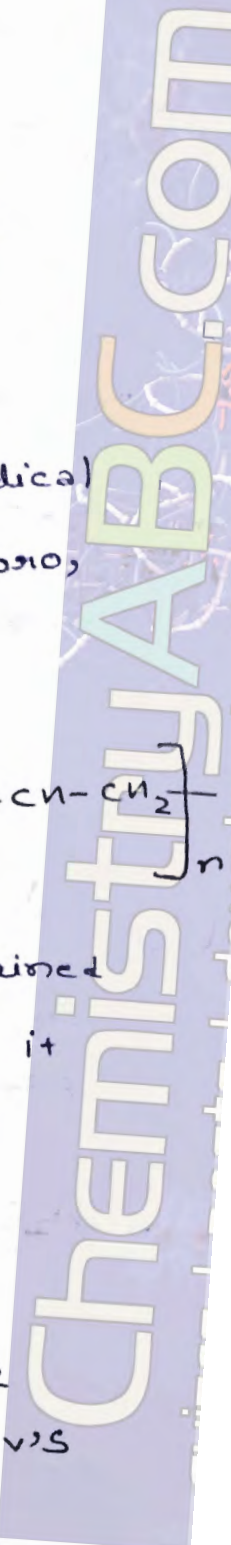
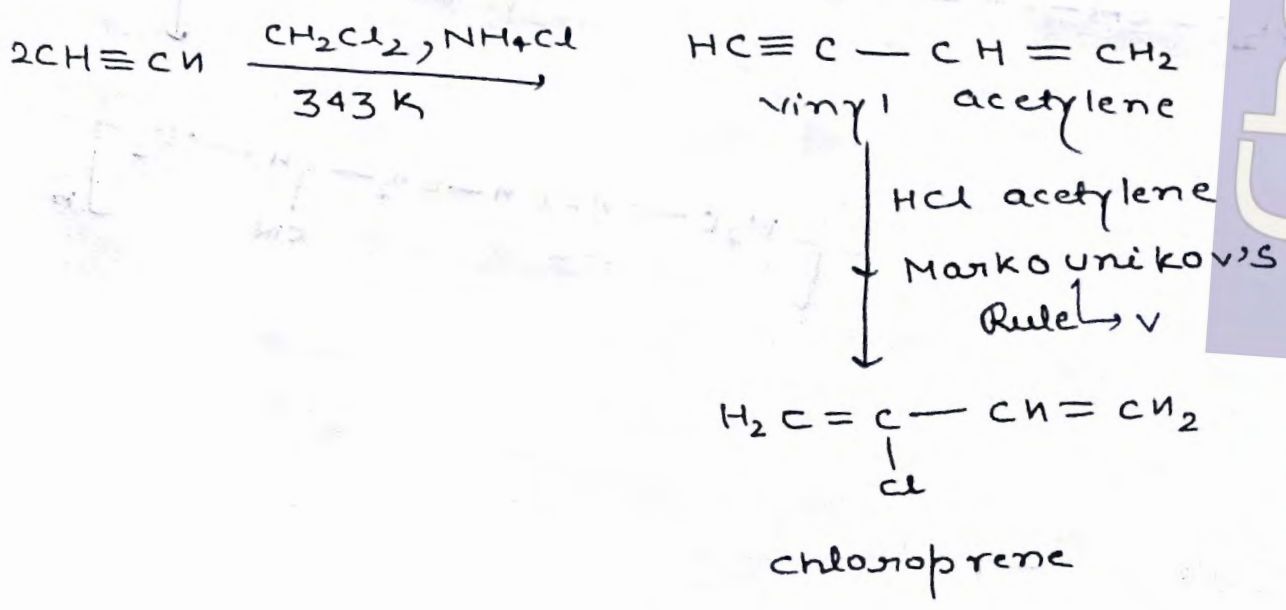
- ① Neoprene
- ② Styrene butadiene rubber (SBR), Buna-S
- ③ Acrylonitrile butadiene rubber, (Buna-N).

It

Neoprene - It is prepared by free radical polymerization of chloroprene (A: (2-chloro, 1,3-butadiene))



Starting material of chloroprene is obtained by dimerization of acetylene by passing it through aq. solution of  $\text{NH}_4\text{Cl}$



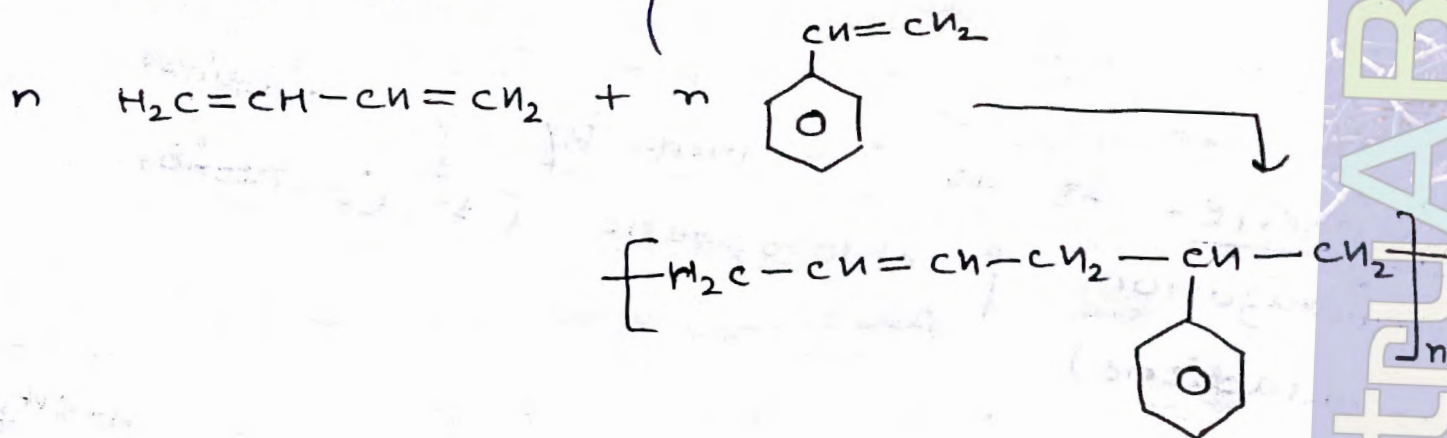


Neoprene is supremely superior to

natural rubber in its stability to its aerial oxidation & its resistance to vegetable oil, mineral oil & other solvent.

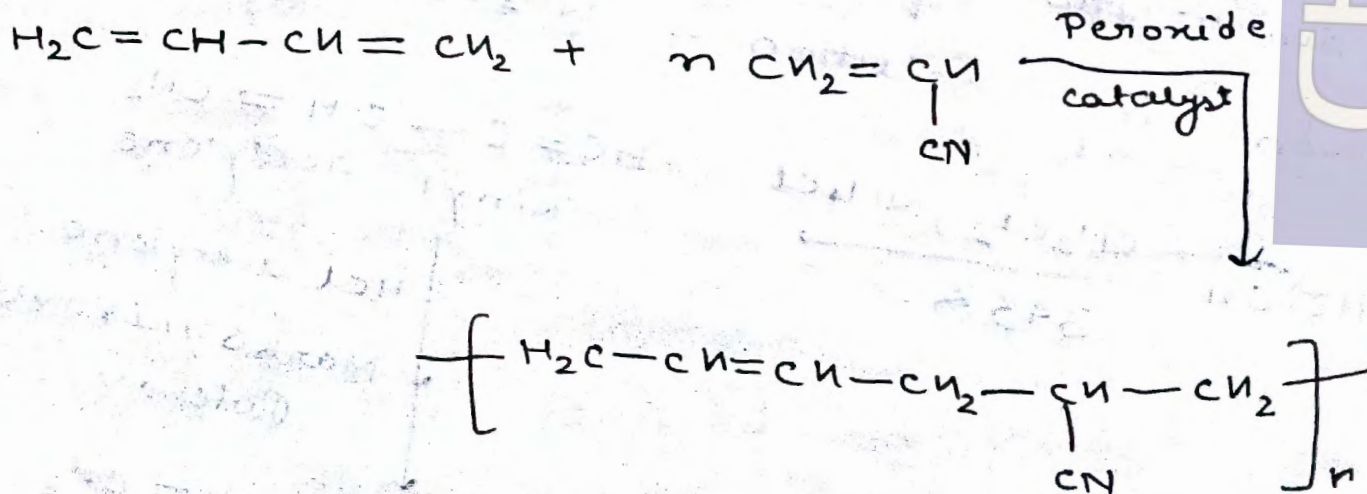
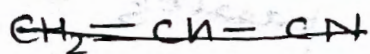
## 2) Buna-S / Styrene butadiene rubber

It is obtained by

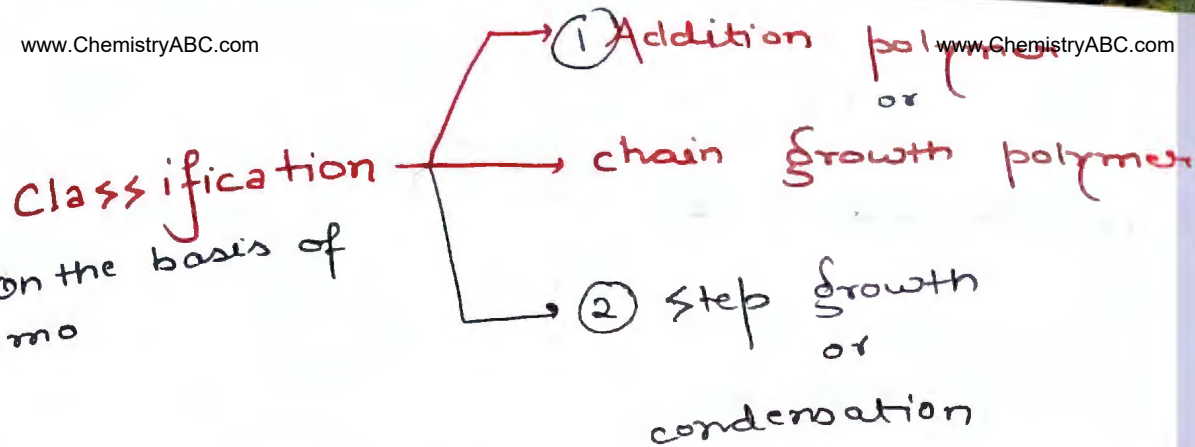


## 3) Acrylonitrile butadiene / Buna-N-

In the presence of peroxide catalyst



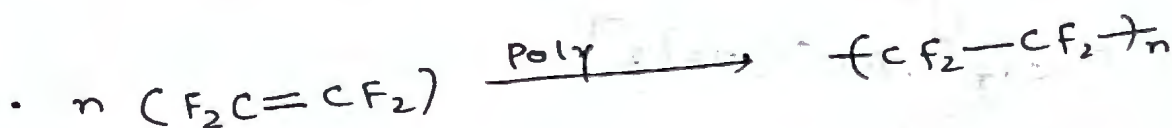
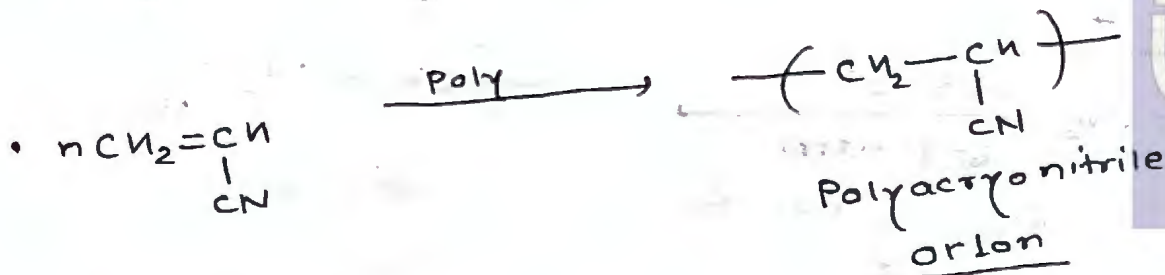
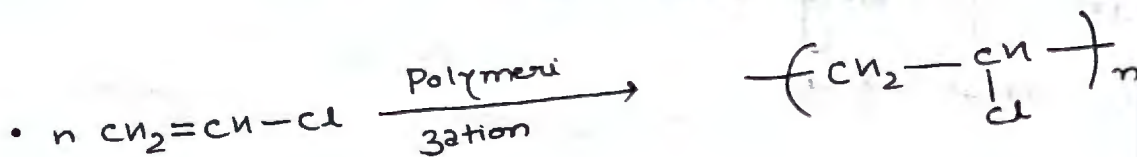
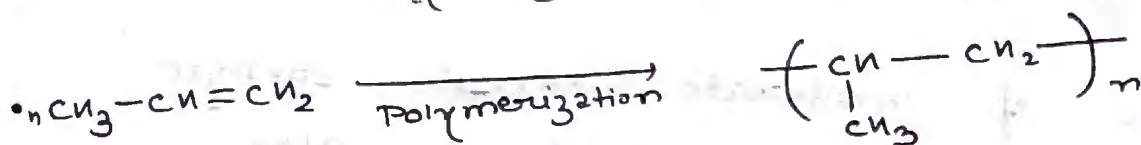
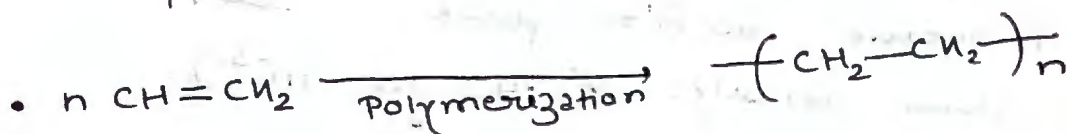
On the basis of mo



### ①. Addition polymer-

formed by direct addition of repeated monomers without elimination of any byproduct is called addition polymer.

In this type monomers are unsaturated compound possessing double or triple bond.





## LDPE $\rightarrow$

@ 350-570K (by ethylene, under high pressure)

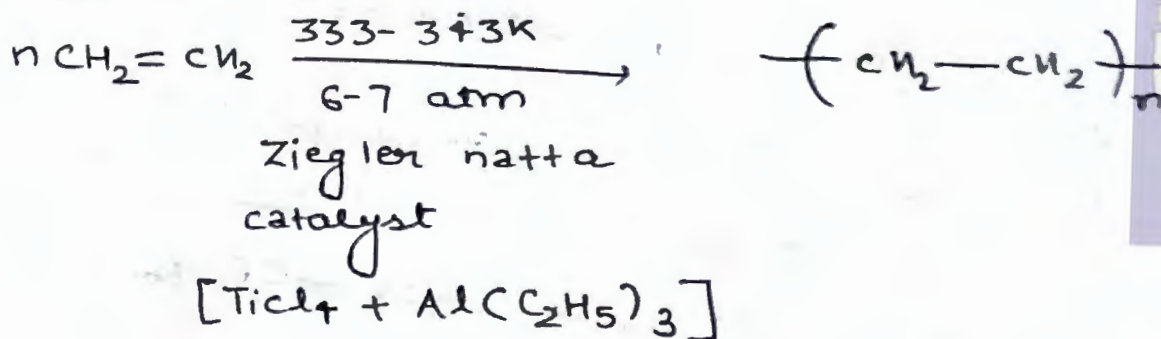
in the presence of traces of oxygen or peroxide initiation, which initiate polymerization.

It consists of highly branched chain molecules.

polyethene molecule don't pack well, so it has low density. ( $0.92 \text{ gm cm}^{-3}$ )

- LDPE of moderate tensile strength.
- used in packing material & also used for insulation

## HDPE $\rightarrow$



comparitively low pressure compared previously beoz of presence of Z.N.C.

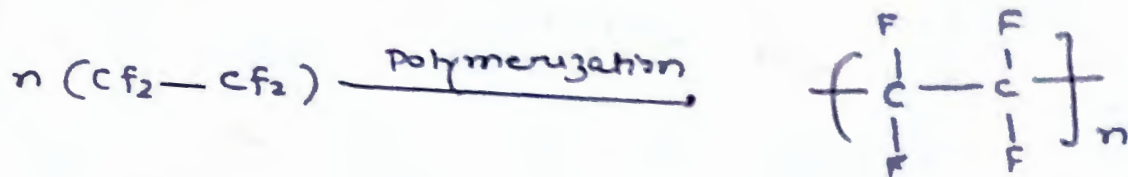
This polymer consists of linear chain & molecule can be closed packed in a space & higher m.p.

& high tensile strength

- Almost unbranched.
- Its chain can be close packed in solid state, such close packing wouldn't be possible in case of branched LDPE. This is the reason why HDPE has higher density than the later.

## PTFE (Poly tetrafluoro ethylene) → Teflon

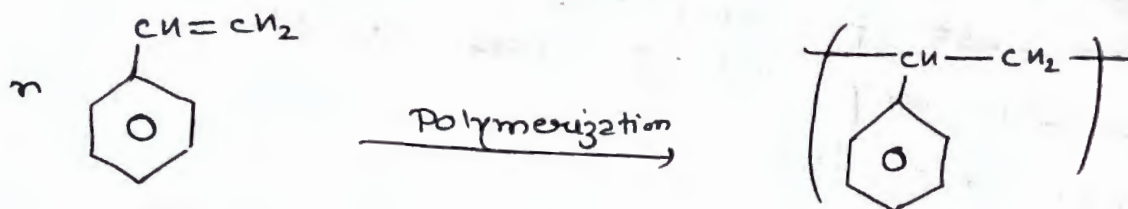
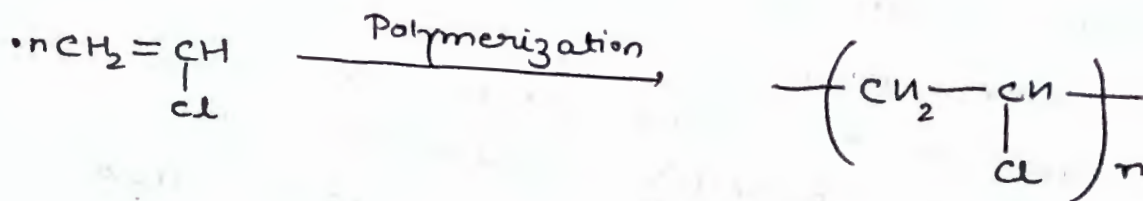
It is an addition polymer of tetra fluoro ethylene



- Inert
- Resistant towards heat & attacked by corrosive reagent such as acid & bases.
- It is bad conductor
- Used in Gaskets, pumps (to avoid leakage), oil seals etc.

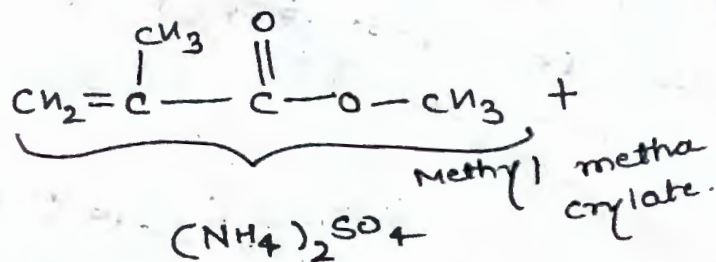
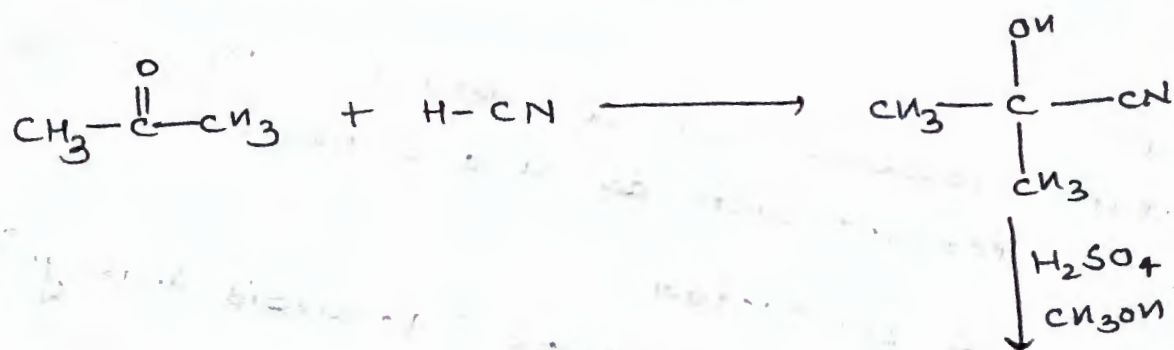
## PAN (Polyacrylonitrile) acrylonitrile →

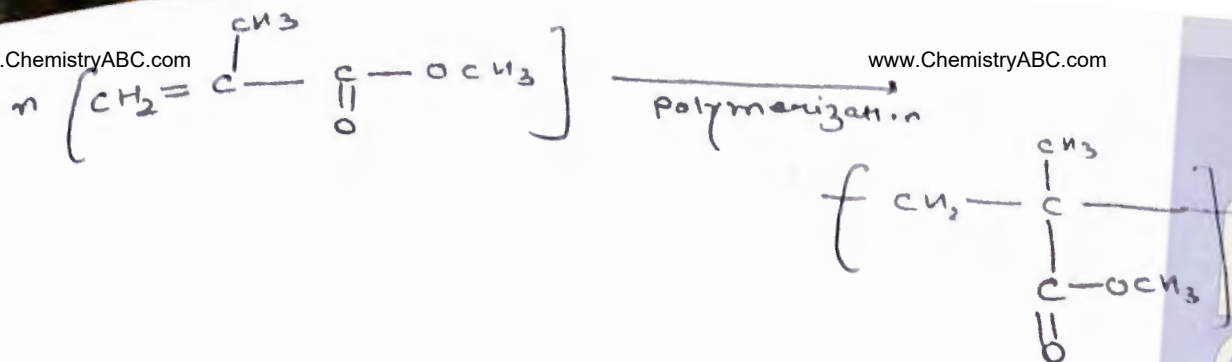
It is obtained by addition polymerization catalyzed by peroxide



## Polymethyl methacrylate -

This is obtained by polymerization of P. MMA (methyl methacrylate). The monomer ethyl is prepared from acetone





used in

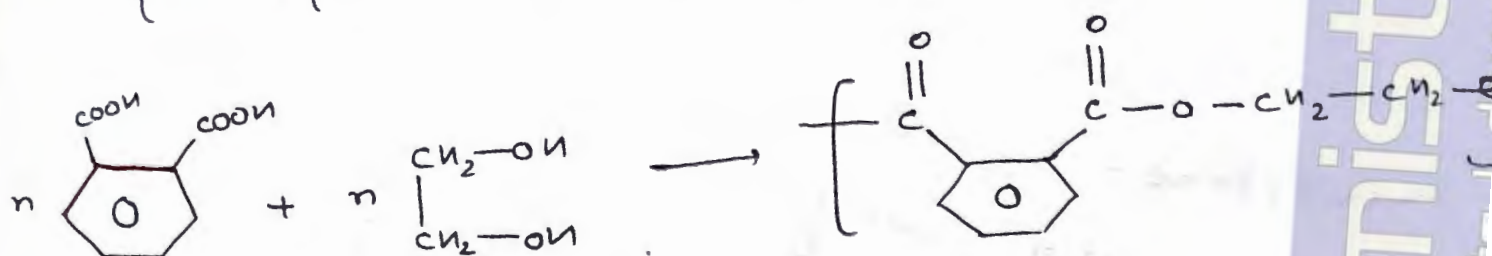
- Aircraft windows, lens, transparent domes.

- Commercial name - Plexiglass.

### Condensation Polymer

Glyptal →

• It is formed by the monomer of ethylene glycol & phthalic acid.



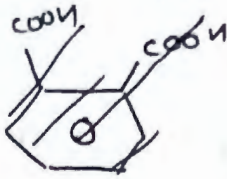
Used in

\* Manufacture of paints & lacquer.  
 A liquid that is spread on wood or metal, that dries to form a hard & shiny surface.

It is used as a protective coating

## Kevlar →

- Monomer - (1,4 phenylene diamine) & tere phthalic acid in a



## Used →

- Making combat helmets.

## Polyesters -

- Contain ester linkages
- These are condensation product of dicarboxylic acid & diols.

e.g. Terylene & Dacron

↳ Polymer of ethylene glycol & tere phthalic acid, obtained by heating a mixture @ 420-460 K in the presence of zinc acetate - antimony trioxide as a catalyst.

- Used in blending with cotton

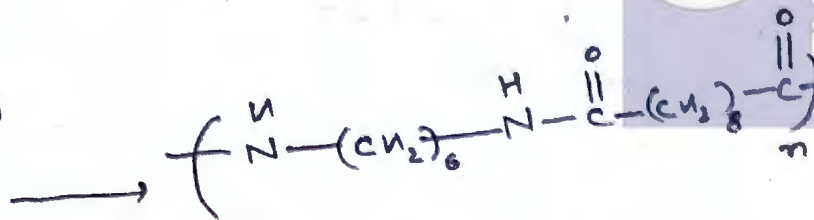
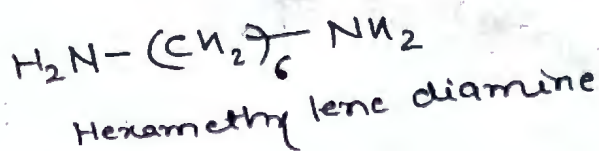
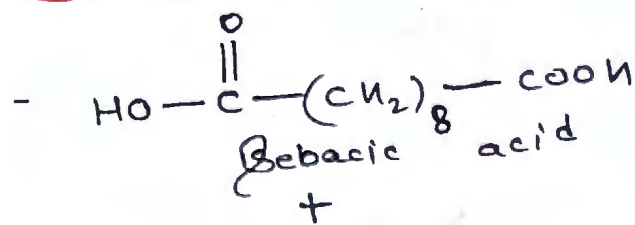
- It is also used in making magnetic recording tapes.
- These polymers have

## Polyamides -

These polymers have amide linkages in chain, these are popularly known as nylon (nylon 66, nylon 6, nylon 6, 10)

- Nylon 66 - Hexamethylene diamine + Adipic acid
- Protein is an example of polyamide. If a protein contains n-amino acids it has (n-1) amide linkages.
- It is used for making bristles & also used in textiles for making seeds.
- It is blended with wool.
- Manufacture of coar cords

### Nylon 6, 10 →



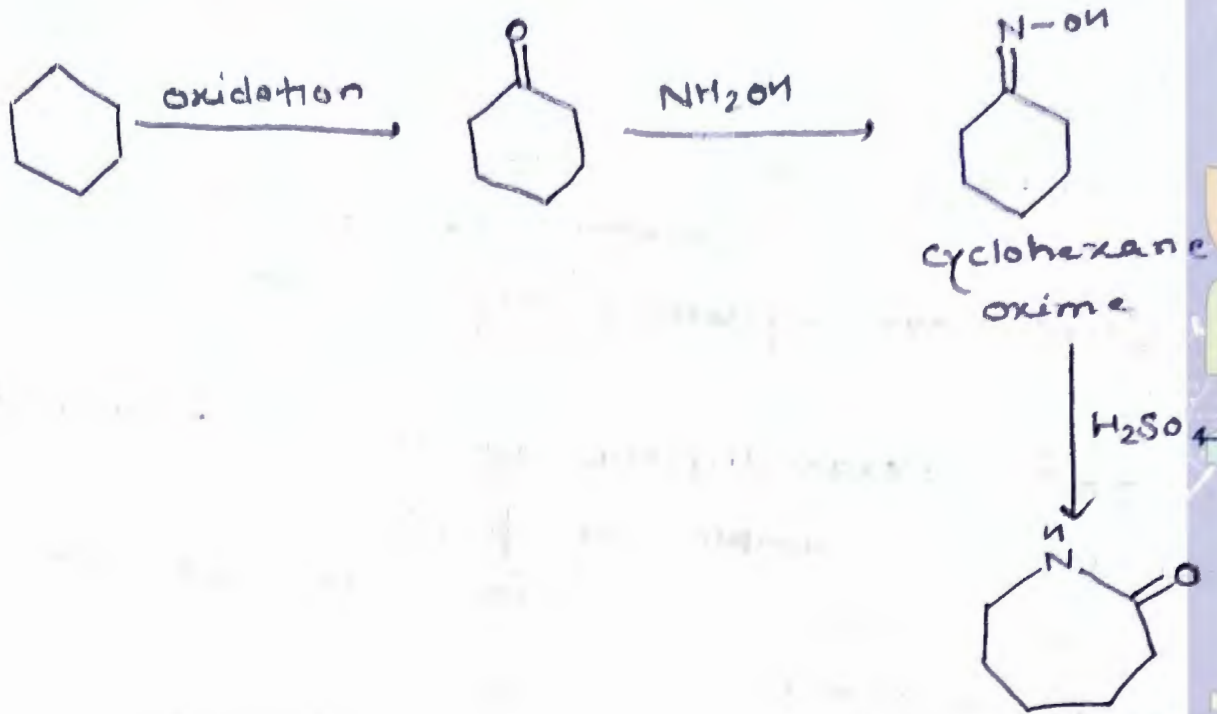
Nylon-6,10

+ nH<sub>2</sub>O

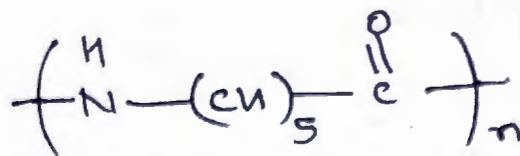
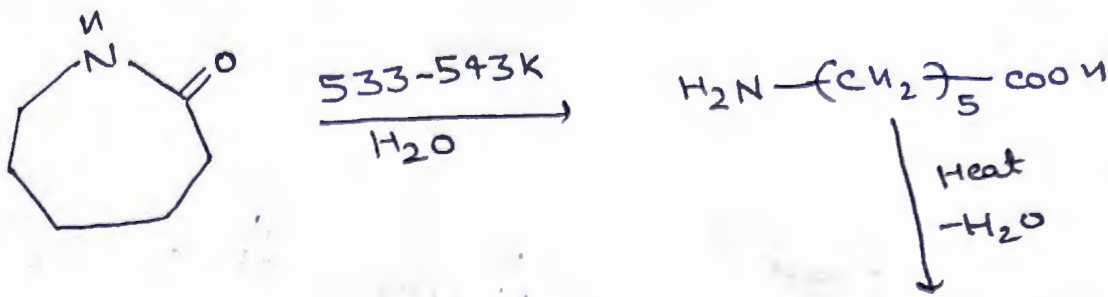


monomer: caprolactum-

Obtaining of caprolactum-



caprolactum on heating with traces of  $\text{H}_2\text{O}$ , gives

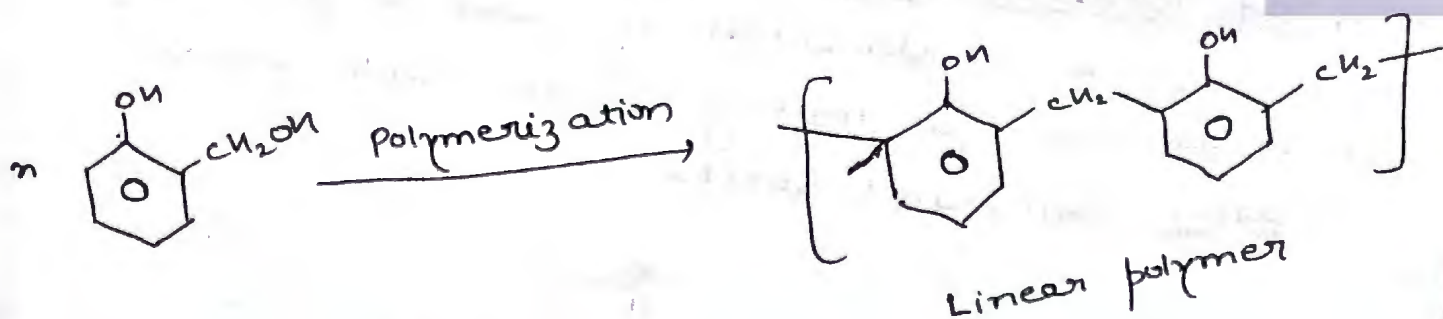
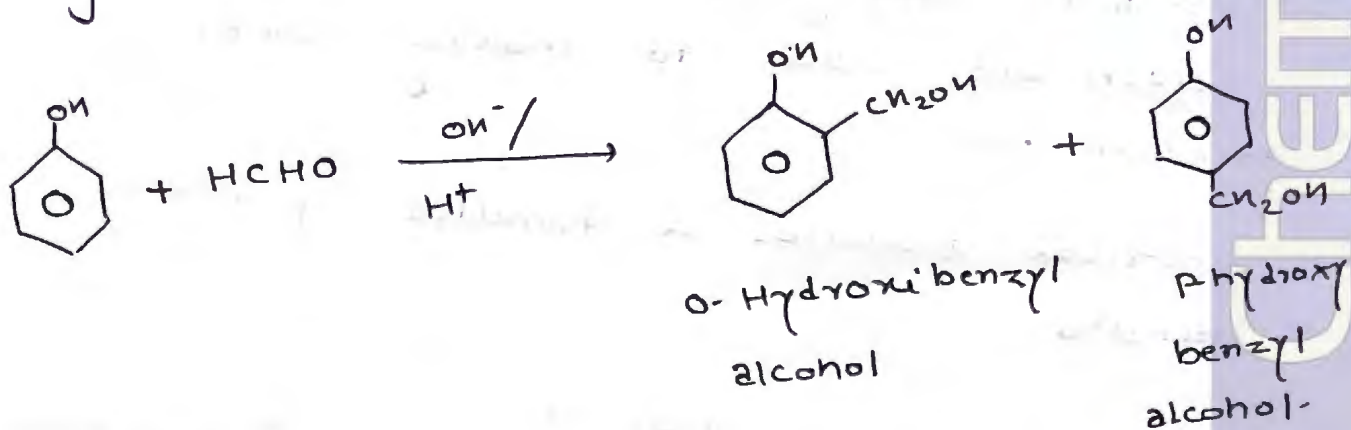


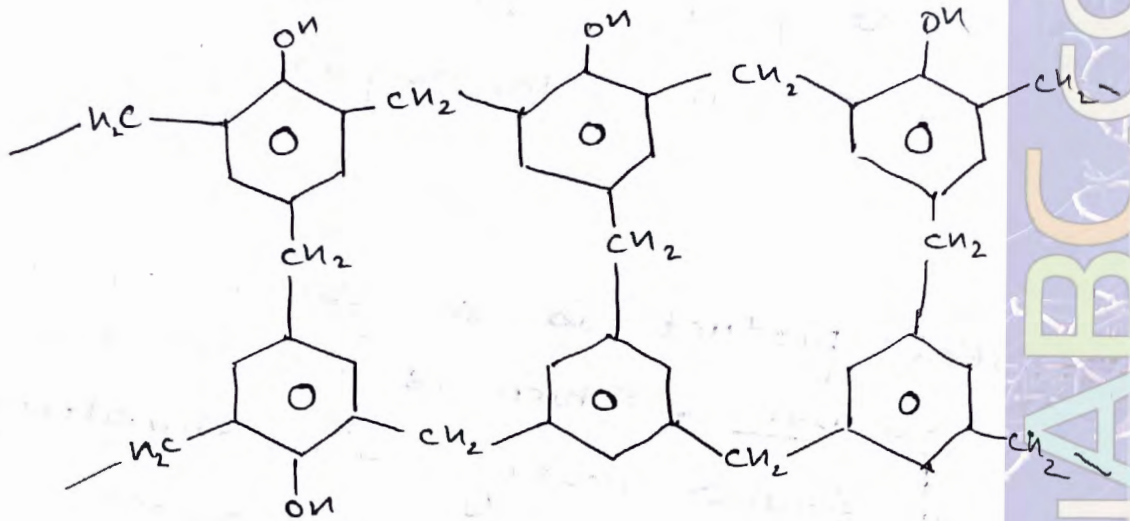
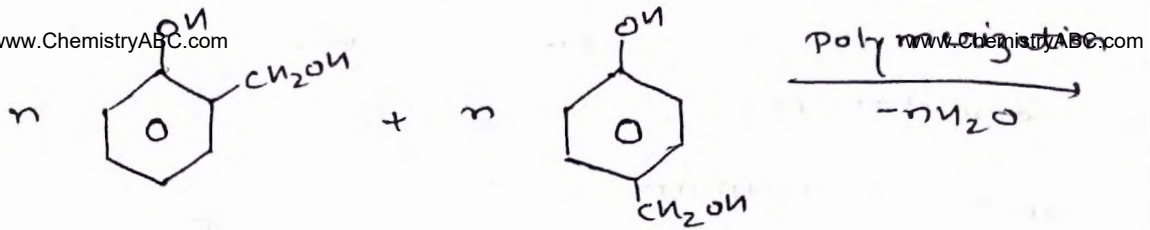
used in manufacture of tyre

# Phenol-formaldehyde resins

- It is a condensation polymer obtained from phenol & formaldehyde
- These reactant or monomer react with each other having methylene bridges either at ortho or at para position or both at ortho/para position with respect to phenolic group.

The initial product is a linear product called Novolac, which is also used in paints on further heating with formaldehyde it undergoes cross linking to form infusible solid called bakelite



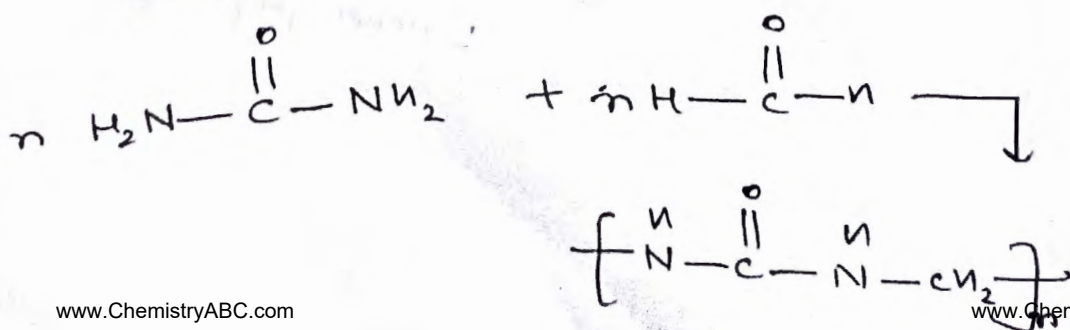


→ HDPE → Give gives hard bakelites which are used to making comb, fountain pen.

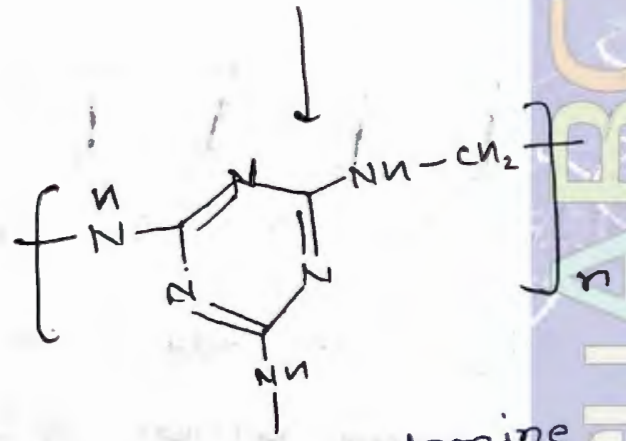
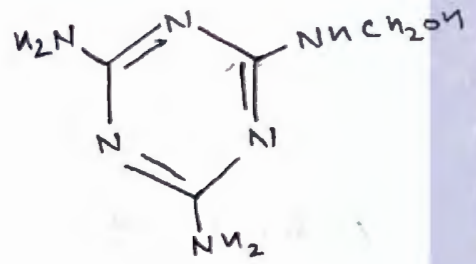
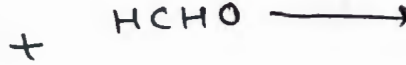
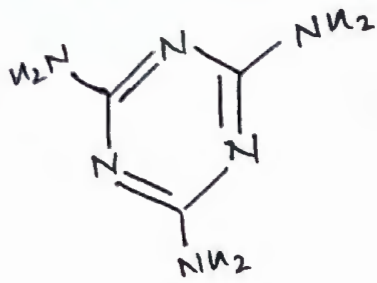
- electrical switches & handles of various utensils.

### Urea formaldehyde resin -

- It is a condensation of urea & formaldehyde
- It is used for making unbreakable cups, plates, laminated sheets.



# Melamine formaldehyde resin -



of melamine

Polymer formed by condensation  
(Heterocyclic triamine) with HCHO.

Used in making crockery.

- Making cup, plates

Q. Buna-S is obtained from polymerization  
of - - - -

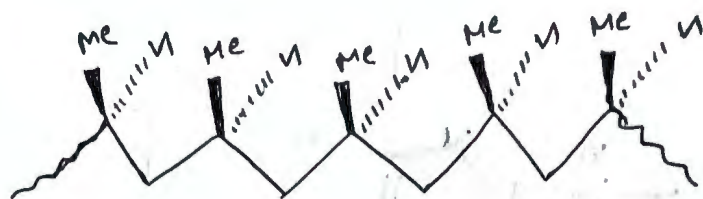
Q. which. to make bullet proofs

- PMMA
- 
- 
- Kevlar

Q. chain growth - polypropylene.

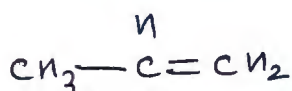
# Classification on the basis of Asymmetric carbon atom

## ① Isotactic →



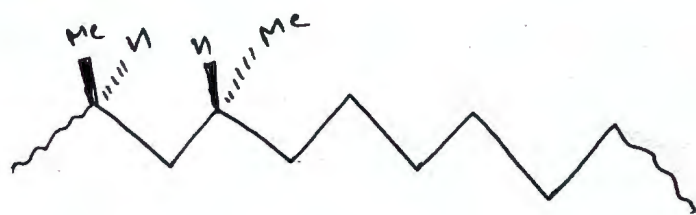
Polymer in which all the carbon atoms have same either D/L are called isotactic polymer.

e.g. Isotactic polypropylene



All the methyl group are located on one side & Hydrogen on other side.

## ② Atactic →



Having random sequence of configuration.

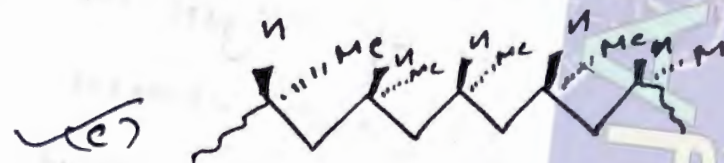
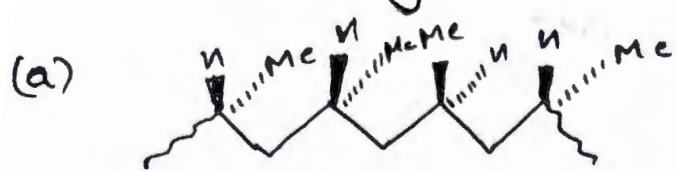
# i) Syndiotactic -

Having regular sequence of configuration.



The substituent group lie alternatively above & below the plane of main chain

Identify the isotactic polypropylene from the following



(b)

(d)

The following polymeric structure is generally referred as -



- (a) Isotactic
- (b) Syndiotactic
- ✓ (c) atactic
- (d) Tactic

Root mean Square end to end distance  
of polymer chain -  
 of a one dimensional flexible chain is  
 given by  $\sqrt{nl^2}$

$$\sqrt{r^2} = \sqrt{nl^2}$$

$n$  = no of segment of the  
 polymer chain.

$l$  = length of each segment or  
 bond length. (C-C)

$$n = \frac{20x}{2} = C_n - 1$$

no of monomer unit.

Q. Average end to end distance  
 of a random coil polymer of  
 $10^6$  monomer in units of segment  
 length is

✓ (a)  $10^{+6}$

(b)  $10^{-5}$

(c)  $10^4$

(d)  $10^3$

$$n = 10^6 - 1$$

$$n = 10^6 - 1$$

$$= 10^6$$

$$\frac{(10^{-6})^2 (1 - 10^{-6})}{10^{-12}}$$

$$\sqrt{r^2} = \sqrt{10^6 \mu^2}$$

$$\sqrt{r^2} = 10^3 \mu$$

Q. What is the end to end distance of a polymer chain  $C_{20}H_{42}$ . Given that c-c bond length is 154 pm

Solution

$$\bullet 154 \times 10^{-12} \text{ m}$$

$$n = 19$$

$$\sqrt{r^2} = \sqrt{19 \times 154 \times 10^{-12}}$$

$$= \sqrt{19} \times 154$$

$$= 4.35 \times 154$$

$$= 671 \text{ pm}$$

Q. A polymer chain has 500 segments, the length of each segment is 1.2 nm. If the polymer chain is totally flexible when dissolved in water. What would be the rms ... (CTIFR 2011)

Ans

$$n = 500$$

$$l = 1.2$$

$$\sqrt{r^2} = \sqrt{nl^2}$$

$$\sqrt{r^2} = \sqrt{500 \times 1.2 \times 1.2}$$

$$1.2 \times \sqrt{500}$$

$$= 1.2 \times 22.360$$

$$26.832$$

(a) 600 nm

(b) 417 nm

(c) 27 nm

(d) 300 nm



# Types of Molar mass →

## ① No average molar mass -

is obtained by carrying out the summation over the fraction of molecule  $\times$  by their corresponding molar mass, multiplied mass.

It is represent by

$$\bar{M}_n = \sum f_i M_i$$

$$f_i = \frac{n_i}{N} \rightarrow \frac{\text{no}}{\text{Total mass NO}}$$

$M_i$  = particular ka mass.

The subscript  $n$  stands for the number & the bar over the symbol  $M_n$  represent the average.

$$f_i \text{ is given by } = \frac{N_i}{N_{\text{total}}}$$

$N_i$  = no of molecule having mass  $M_i$   
 $N_{\text{total}}$  total no of molecule

$$\bar{M}_n = \frac{\sum N_i M_i}{N_{\text{total}}}$$

Q 3 types of molecules containing no of molecule  $n_1, n_2$  &  $n_3$  & their corresponding molar mass are  $m_1, m_2, m_3$  then  $\bar{M}_n$

$$\bar{M}_n = \frac{N_1 M_1 + N_2 M_2 + N_3 M_3}{N_1 + N_2 + N_3}$$

The method used to determine the no average molar mass is osmometry (osmotic pressure method).

(A)- Osmotic pressure for the measurement of molar mass

$$\frac{\pi}{cRT} = \frac{1}{M_n} + Bc$$

, virial equation.

or

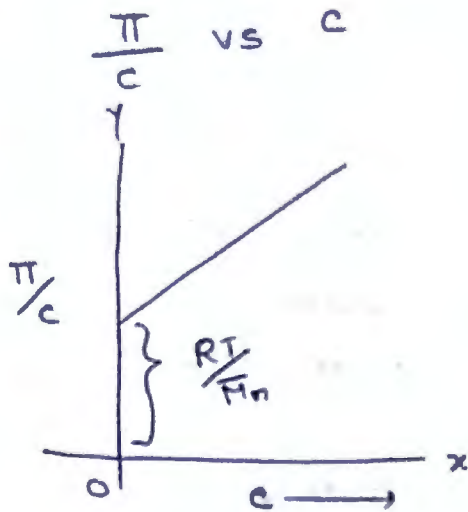
↳ second virial coefficient

$$\frac{\pi}{c} = \frac{RT}{M_n} + Bc \cdot RT$$

$c$  = Polymer concentration

$\frac{\pi}{c}$  = reduce osmotic pressure.

On plotting a graph between



Q. The osmotic pressure of polymeric sample @ different concentration ( $c$ ) was measured at  $T(K)$ . A plot of  $\pi/c$  versus  $c$  gave a straight line with slope ( $m$ ) & intercept  $c'$ .  
The no average mol. wt of the polymer is ( $R =$  Gas constant).

Solution -

$$\frac{\pi}{c} = \frac{1}{M_n} + Bc$$

(a)  $\frac{RT}{c'}$

$$c' = \frac{RT}{M_n}$$

(b)  $c'/RT$

$$\Rightarrow M_n = \frac{RT}{c'}$$

(c)  $RT$

(d)  $MRT$

Q. A plot of osmotic pressure against concentration (g/L) of a polymer is constructed. The slope of graph

- ✓ (a) ↑ with ↑ in Temperature
- (b) ↑ with ↑ in molar mass of polymer.
- × (c) ↑ with ↓ in conc of polymer.
- (d) ↓ with ↑ in temperature.

Q →

② Mass average / Weight average →  
( $\bar{M}_m$ ) ( $\bar{M}_w$ )  
molar mass

The mass average is obtained by carrying out the summation over the mass fraction  $\times$  by corresponding molar mass

$$W_i = \frac{m_i}{M_{\text{total}}}$$

mass fraction

$$\bar{M}_w = \bar{M}_m = \sum W_i M_i$$

$$\bar{M}_w = \frac{\sum m_i M_i}{m_{\text{total}}}$$

$m_i$  = mass of the polymer molecule  
having mass  $M_i$

$M_{\text{total}}$  = total mass of the  
sample of the polymer.

which is given by

$$\begin{aligned} \bar{M}_w &= \frac{\sum m_i M_i}{M_{\text{total}}} \\ &= \frac{\sum \frac{m_i}{M_i} \times M_i \times M_i}{\sum m_i \times \frac{M_i}{M_i}} \\ &= \frac{\sum N_i M_i^2}{\sum N_i M_i} \end{aligned}$$

Molecule

$$\frac{N_i}{N_A} = \frac{m_i}{M_i}$$

Avogadro

$$N_i = N_A \times \frac{m_i}{M_i}$$

$$\frac{N_i}{N_A} = \text{mole}$$

The method used for determination of mass average  
average  
Light scattering method

### 3) Z average molar mass

$$\bar{M}_z = \frac{\sum N_i M_i^3}{\sum N_i M_i^2}$$

The method used for determining Z average molar mass is sedimentation method

### 4) Viscosity Average molar mass

$$\bar{M}_v = \left( \frac{\sum N_i M_i^{a+1}}{\sum N_i M_i} \right)^{\frac{1}{a}}$$

The value of  $a$  lies in the range  $0.5 < a < 1.0$

when  $a = 1$

$$\bar{M}_v = \bar{M}_m$$

The constant  $a$  is known as Mark-Houwink <sup>wink</sup> exponent.

method used - Intrinsic viscosity method.

$$[\eta] =$$

$[\eta]$  = intrinsic viscosity

$c$  = concentration of polymeric solution

$\eta$  = viscosity of solution

$\eta_0$  = viscosity of pure solvent

$$[\eta] = \lim_{c \rightarrow 0} \frac{1}{c} \left[ \frac{\eta}{\eta_0} - 1 \right]$$

## Mark Houwink equation -

The intrinsic viscosity of polymeric solution is found to increase, with increase in average molar mass of the polymer, this dependence is expressed as,

$$[\eta] = K \cdot \bar{M}_v^a \quad \rightarrow \textcircled{1}$$

Mark Houwink eq.

$K$  &  $a$   $\rightarrow$  are known as Mark Houwink eq.

Q  $\rightarrow$  Using standard equation  $[\eta] = K \cdot \bar{M}_v^a$  for a solution of polymer & any information from the graph, in identify viscosity average molar mass ( $\bar{M}_v$ )

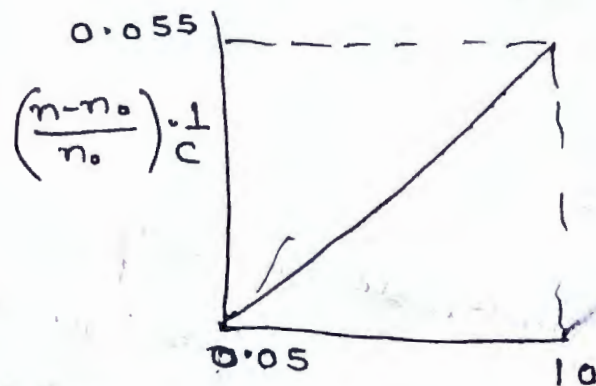
[given that  $a = 0.5$ ,  $K = 5 \times 10^{-5} \text{ Lg}^{-1}$ ]

(a)  $10^3 \text{ g mol}^{-1}$

(b)  $10^4 \text{ g mol}^{-1}$

(c)  $10^5 \text{ g mol}^{-1}$

(d)  $10^6 \text{ g mol}^{-1}$



[ June, 2015 ]

We analyze

$$[\eta] = 0.05$$

$$0.05 = 5 \times 10^{-5} [\bar{M}_v]^{0.5}$$

$$[\bar{M}_v]^{\frac{1}{2}} = \frac{0.05}{5 \times 10^{-5}} = \frac{5 \times 10^{-2}}{5 \times 10^{-5}} = 10^{-2+5} = 10^3$$

$$[\bar{M}_v] = 10^6$$

## Polydispersity index (PDI) →

$$* \quad \bar{M}_z \left\{ \begin{array}{l} \text{or} \\ = \end{array} \right. \bar{M}_w \geq \bar{M}_v > \bar{M}_n$$

The ratio of

$$\text{P.D.I.} = \frac{\bar{M}_w}{\bar{M}_n}$$

~~z w v n~~  
z w v n

The ratio of  $\bar{M}_w / \bar{M}_n$

$$\therefore \bar{M}_w > \bar{M}_n$$

$$\Rightarrow \text{P.D.I. is } > 1$$

if in any case  $\bar{M}_w = \bar{M}_n$

$$\Rightarrow \text{P.D.I.} = 1 \text{ (monodisperse)}$$

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Q- for a polydisperse macromolecular colloid osmometry gives,

- (a) weight - average
- ✓ (b) Number- average
- (c) both weight average & number average
- (d) viscosity average molecular wt.

Q- There are several types of mean molar masses of polymer & they are dependent on experimental method

- ① osmometry
- ② Light scattering
- ③ Sedimentation

Correct relation between mean molar masses & experimental method is-

(a)  $\bar{M}_n \Leftrightarrow (3)$ ,  $\bar{M}_w \Leftrightarrow (2)$ ,  $\bar{M}_z \Leftrightarrow (1)$

(b)  $\bar{M}_n \Leftrightarrow (2)$ ,  $\bar{M}_w \Leftrightarrow (3)$ ,  $\bar{M}_z \Leftrightarrow (1)$

✓ (c)  $\bar{M}_n \Leftrightarrow (1)$ ,  $\bar{M}_w \Leftrightarrow (2)$ ,  $\bar{M}_z \Leftrightarrow (3)$

(d)  $\bar{M}_n \Leftrightarrow 1$ ,  $\bar{M}_w \Leftrightarrow 3$ ,  $\bar{M}_z \Leftrightarrow (2)$

- Q- The no average molar mass  $\bar{M}_n$ , wt average molar mass ( $\bar{M}_w$ ) of a polymer are obtained respectively
- (a) Osmometry & light scattering  
 (b) Osmometry viscosity measurement  
 (c) Light scattering & sedimentation m  
 (d) Viscosity & light scattering

2  
 Q- A polymer sample has the following composition

No of molecules	Molecular wt
10	1000
50	2000
40	4000

The P.D.I. of the polymer is

(a)  $\frac{850000}{27}$

(b)  $\frac{85}{81}$

(c)  $\frac{850}{729}$

(d)  $\frac{729}{850}$

$$\bar{M}_w = \frac{10 \times 1000 + 50 \times 2000 + 40 \times 4000}{100}$$

$$\bar{M}_n = \frac{10,000 + 10,000 + 16,000}{100}$$

$$\bar{M}_n = \frac{270000}{100} = 2700$$

(33)

$$\bar{M}_w = \frac{\sum N_i M_i^2}{\sum N_i M_i}$$

$$= \frac{10 \times 10^6 + 50 \times 4 \times 10^6 + 16 \times 10^6}{270000}$$

$$= \frac{850 \times 10^6}{270000} = \frac{850 \times 10^2}{27}$$

$$P.D. \cdot I = \frac{\frac{850 \times 10^2}{27}}{2700} = \frac{850}{729}$$

The ratio of

Q. The ratio of mean molar masses of a given polymer sample has determined by light scattering, sedimentation, osmotic pressure, measurement

- (a) 112
- (b) 212
- (c) 121
- (d) 221

Q. Which of the following method yield a rate average <sup>molecular</sup> wt m of polymer.

- (a) vapour pressure osmometry
- (b) membrane "
- (c) Light scattering
- (d) mass spectrometry

Q. The molar masses of mono disperse & poly disperse polymer obey respectively the condition-

$$\bar{M}_v = \text{No average molecular weight}$$

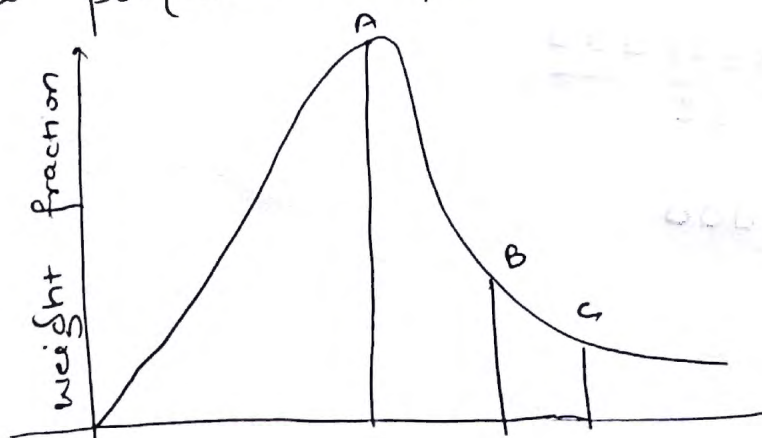
$$\bar{M}_w = \text{Weight " " " "}$$

- (a)  $\bar{M}_n > \bar{M}_w$  &  $\bar{M}_n < \bar{M}_w$
- (b)  $\bar{M}_n = \bar{M}_w$  &  $\bar{M}_n < \bar{M}_w$
- (c)  $\bar{M}_n \neq$
- (d)

Q. The mark How-Houwink equation  $[\eta] = K M^a$  is used for determination of

- (a) No average
- (b)
- (c) viscosity average molar mass (✓)
- (d)

Q. The distribution of molar mass in typical polymer sample



(a)  $\bar{M}_w$ ,  $\bar{M}_v$  &  $\bar{M}_n$  respectively

(b)  $\bar{M}_n$ ,  $\bar{M}_v$  &  $\bar{M}_w$  "

(c)  $\bar{M}_v$ ,  $\bar{M}_w$  &  $\bar{M}_n$  "

(d)  $\bar{M}_n$ ,  $\bar{M}_w$  &  $\bar{M}_v$  "

Q. A polydisperse polymer has 10 molecules of molar mass  $20,000 \text{ g mol}^{-1}$  & fifteen molecules of molar mass  $10,000 \text{ g mol}^{-1}$

The no average molar mass in  $\text{g mol}^{-1}$

(a) 13,000

(b) 14,000

(c) 15,000

(d) 16,000

$$\bar{M}_n = \frac{N_1 M_1 + N_2 M_2}{N_1 + N_2}$$

$$\frac{10 \times 20,000 + 15 \times 10,000}{25}$$

$$= \frac{3,50,000}{25}$$

$$= 14,000$$

Q. for a monodisperse polymer  $\bar{M}_n$  &  $\bar{M}_w$  are related acc to [www.ChemistryABC.com](http://www.ChemistryABC.com)

- (a)  $\bar{M}_w < \bar{M}_n$
- (b)  $\bar{M}_w = \bar{M}_n$
- (c)  $\bar{M}_w > \bar{M}_n$
- (d)  $\bar{M}_w < \log \bar{M}_n$

53. The correct expression for the product  $(\bar{M}_n \cdot \bar{M}_w)$  [ $\bar{M}_n$  &  $\bar{M}_w$  are the number average & weight average molar masses, respectively, of a polymer] is Dec-2015

- (1).  $N^{-1} \sum_i N_i M_i$
- (2).  $N^{-1} \sum_i N_i M_i^2$
- (3).  $N / \sum_i N_i M_i$
- (4).  $N / \sum_i N_i M_i^2$

$$\bar{M}_n = \frac{\sum N_i M_i}{N} \rightarrow \textcircled{2}$$

$$\bar{M}_w = \frac{\sum m_i M_i}{M_{total}}$$

$$= \frac{\sum N_i M_i^2}{\sum N_i M_i} \rightarrow \textcircled{1}$$

$$\textcircled{1} \times \textcircled{2}$$

$$\frac{\sum N_i M_i}{N} \times \frac{\sum N_i M_i^2}{\sum N_i M_i}$$

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