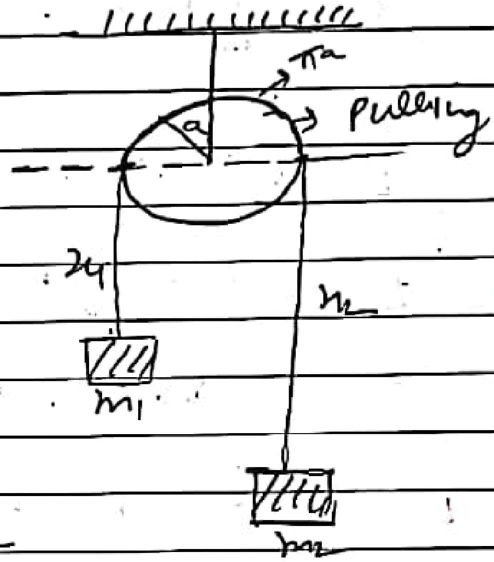
m-Imp

* At woods machine :-



Let us consider two masses m_1 & m_2 which are hanging on the wire by a pulley. And pulley is friction less.
And there are two coordinate x_1 & x_2 the motion of these particle by ~~cons~~ constraint. Condition can be written as.

$$L = x_1 + x_2 + \pi a$$

$$x_2 = L - x_1 - \pi a \quad \text{--- (1)}$$

diff. w.r.to t

$$\dot{x}_2 = -\dot{x}_1 \quad \text{--- (2)}$$

Where L is the length of the wire and a is the Radius of pulley

Now the K.E of both masses can be written as.

$$T = \frac{1}{2}mv^2 = \frac{1}{2}m_1\dot{x}_1^2 + \frac{1}{2}m_2\dot{x}_2^2 \quad \text{--- (3)}$$

$$V = V_1 + V_2$$

due to the gravitational force the P.E of both masses should be -ve.

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$$V = -m_1 g x_1 - m_2 g x_2 \quad \text{--- (ii)}$$

we know that

$$L = T - V$$

$$L = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}_2^2 + m_1 g x_1 + m_2 g x_2$$

from eq (i)

$$L = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}_1^2 + m_1 g x_1 + m_2 g (L - x_1 - \pi a)$$

$$L = \frac{1}{2} \dot{x}_1^2 [m_1 + m_2] + g x_1 (m_1 - m_2) + m_2 g (L - \pi a)$$

$$V_0 = -m_2 g (L - \pi a)$$

$$L = \frac{1}{2} \dot{x}_1^2 [m_1 + m_2] + g x_1 (m_1 - m_2) + V_0$$

Now from the Lagrange Hamilton eq:

$$\frac{\partial L}{\partial x_1} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) = 0$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) - \frac{\partial L}{\partial x_1} = 0$$

diff w.r.to x_1 and $\dot{x}_1$ we get

$$\frac{\partial L}{\partial x_1} = g(m_1 - m_2)$$

$$\frac{\partial L}{\partial \dot{x}_1} = \dot{x}_1 (m_1 + m_2)$$

$$\frac{d}{dt}\left[\dot{x}_1 (m_1 + m_2) - g(m_1 - m_2)\right] = 0$$

$$\ddot{x}_1 (m_1 + m_2) - g(m_1 - m_2) = 0$$

$$\ddot{x}_1 (m_1 + m_2) = g(m_1 - m_2)$$

$$\boxed{\ddot{x} = \frac{g(m_1 - m_2)}{m_1 + m_2}}$$

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Imp

* D Alemberts principle :-

This principle is based upon the principle of virtual work.

i.e

$$W = \sum_{i}^{N} \vec{F_i} \cdot \delta \vec{r_i} = 0 \quad \text{--- (1)}$$

If there is small N particle then each particle exert the force $F_1, F_2 \ldots F_N$ and gets the displacements $\delta r_1, \delta r_2, \ldots \delta \vec{r_n}$ if the particle is restricted i.e the constraints condition is applied on a particle then the total force of the body is equal to the sum of applied force

$$\vec{F_i} = \vec{F_{ia}} + f_i \quad \text{--- (2)}$$

[F_{ia} = applied force
f_i = constraints force]

Now we know that the conservative force is always applied when the displacement is restricted i.e. the constraints force is ⊥ at to the the displacement

$$\sum_{i}^{N} \vec{F_{ia}} \cdot \delta r_i = 0 \quad \text{--- (3)}$$

The equation is termed as virtual work.

To interpret to equalibrium system D Alembert adopted an idea of the reversed force

He confess that that a system will remain in equalibrium under the action of a force equal to the

Actual force F_i And Reverse Effective force $\dot{p}_i$

$$\vec{F_i} = \frac{dp_i}{dt} = \dot{p}_i$$

$$\vec{F_i} = \vec{\dot{p}_i}$$

$$\vec{F_i} - \dot{p}_i = 0$$

$$\sum_i^N (\vec{F_i} - \dot{p}_i) \cdot \delta \vec{r_i} = 0$$

$$\sum_i^N (\vec{F_{ia}} - \dot{p}_i) \cdot \delta r_i + \sum_i^N \vec{f_i} \cdot \delta \vec{r_i} = 0$$

since force of constraints are no more in picture, so it is better to drop.

$$\boxed{\sum_i (\vec{F_i} - \dot{p}_i) \cdot \delta \vec{r_i} = 0}$$

which is called D-Alembert Principle which shows that the Reverse Effective force always decrease the value of Applied force