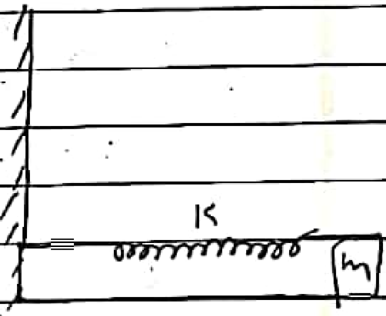
* Application of Lagrangian Eq by variation principle.

1. Liniear Harmonic ossilation :-

Let us consider a particle of mass m when it is attached at a one fix point with a spring and force K.



Now the spring displaced from n. Equalibrium position then the spring moves from one point to another point from the fixed point the K.E. can be written as.

$$T = \frac{1}{2} m v^2$$

$$= \frac{1}{2} m \dot{x}^2 \quad \text{—— (1)}$$

Now from hooks law we know that the force is directly proportional to $\propto x$

$$F \propto x$$

$$F = -Kx \quad \text{—— (2)}$$

we also know that the -ve gradient of potential Energy is equal to conservative force.

$$F = -\text{grad } V$$

$$F = -\frac{dV}{dx} \quad \text{—— (3)}$$

$$Kx = \frac{dV}{dx}$$

$$dV = Kx\,dx$$

$$\int dV = K \int x\,dx$$

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On Integrating, Now

$$V = \frac{1}{2}Kx^2 + C$$

where C is Integral constants.

$V = 0, x = 0$ [Boundary Condition]

then $\boxed{C = 0}$

$$V = \frac{1}{2}Kx^2 \quad \text{--- (4)}$$

$$L = T - V$$

$$L = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}Kx^2 \quad \text{--- (5)}$$

also from Hamilton Lagrangian Eq

we know that

$$\frac{\partial L}{\partial x} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = 0$$

diff w.r.to x And $\dot{x}$ of Eq (5).

$$\frac{\partial L}{\partial x} = -Kx$$

$$\frac{\partial L}{\partial \dot{x}} = m\dot{x}$$

$$-Kx - \frac{d}{dt}(m\dot{x}) = 0$$

$$\cancel{-Kx - m\ddot{x} = 0}$$

$$Kx + m\ddot{x} = 0$$

$$\ddot{x} + \frac{K}{m}x = 0$$

from Simple Harmonic Condition

$$\ddot{x} + w^2 x = 0$$

$$w^2 = \frac{K}{m}$$

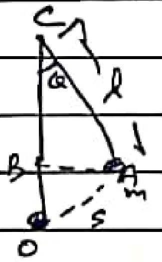
$$w = \sqrt{\frac{K}{m}}$$

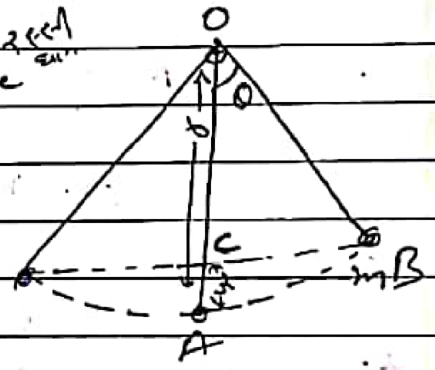
we know that

$$\boxed{T = \frac{2\pi}{w}\sqrt{\frac{g}{l}}} \quad \boxed{T = 2\pi\sqrt{\frac{m}{k}}}$$

This is the K·E Equation of Harmonic ossilation which depends upon the mass Condition and Spring constants K

2. Simple pendulam :-





Let us consider a ~~plane~~ of length l in which a particle of mass m is attached in one side and another is fix at point o.

Now the particle of mass m is rotated about a fixed point O then the K·E can be written as

$$T = \frac{1}{2} m v^2$$
$$= \frac{1}{2} m (l \dot{\theta})^2$$
$$T = \frac{1}{2} m l^2 \dot{\theta}^2 \quad -(1)$$

$\theta = \frac{s}{l}$

$s = l\theta$

$T = \frac{1}{2} m v^2$

$v = \frac{ds}{dt}$

$\frac{d}{dt}(l\theta)$

$l\frac{d\theta}{dt}$

$l\dot{\theta}$

And potential about fix point

$$V = mgh = mg(AO - OC)$$
$$V = mgl - mgl\cos\theta \quad -(2)$$

$$L = T - V$$
$$= \frac{1}{2} m l^2 \dot{\theta}^2 - mgl(1 - \cos\theta)$$

diff. w. r.to θ And $\dot{\theta}$

$$\frac{\partial L}{\partial \theta} = -mgl \sin\theta$$

$$\frac{dL}{d\dot{\theta}} = ml^2 \dot{\theta}$$

Now from Lagrangian Hamilton Eq we know that

$$\frac{\partial L}{\partial \theta} - \frac{d}{dt}\left(\frac{dL}{d\dot{\theta}}\right) = 0$$

$$-mgl \sin\theta - \frac{d}{dt}(ml^2\dot{\theta})$$

$$mgl\sin\theta + ml^2\ddot{\theta} = 0$$

from the simple pendulam $\sin\theta = \theta$ become use θ is very small

$$mgl\theta + ml^2\ddot{\theta} = 0$$

$$ml(g\theta + l\ddot{\theta}) = 0$$

$$\ddot{\theta} + \frac{g\theta}{l} = 0$$

from simple Harmonic motion Eq.

$$\ddot{\theta} + w^2\theta = 0$$

$$w = \sqrt{\frac{g}{l}}$$

$$\boxed{T = 2\pi\sqrt{\frac{l}{g}}}$$

This is the equation Expression for time period for simple pendulam.

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