Constraints:— The Restriction or boundation which is applied on a particle to restrict it's motion is called Constraints

Or

The Restriction or boundation which is Applied on a particle to restrict the motion of the particle in a particular path is called Constraints

Case I- When a particle moves on the circumference of the circle then the Equation of motion of the particle can be Expressed in the form of

$$x^2+y^2=r^2$$

Case II-

In the case of the sphere the Equation of motion can be Expressed in the form of

$$x^2+y^2+z^2=r^2$$

Holonomic and Non holonomic Constraints

If the Constraints can be expressed in the form of

$f(x_1, y_1, z_1, x_2, y_2, z_2 - - t) = 0$ is called holonomic Constraints

And if the Equation of motion of the Constraint Condition

$f(x_1, y_1, z_1, x_2, y_2, z_2 \cdots t) \neq 0$

can not satisfy the above Equation is called the Non holonomic Constraints

In other words we can say that if a particle in not holonomic then it is Non-holonomic.

Rhenomic And Scleronomic Constraints →

When the motion of the particle depend upon the time component then the constraint condition is formed is known as Rhenomic Constraints and if it does not depend on the time component then it is known as Scleronomic Constraints.

Rhenomic Constraints -

1. Time dependent
2. The force is almost non conservative in nature
3. The total work done by the rhenomic constraint is not equal to zero ($w \neq 0$)
4. Pendulum of variable length

Scleronomic Constraints

Time independent

The force is almost conservative in nature

The total work done by the scleronomic constraint equal to zero $w = 0$

Pendulum of fixed length


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(Hamilton Variation Principle)

Hamilton Variation Principle: This is also Integral principle. Consider the motion of system in configuration space b/w two given time. Statement: the motion of a system b/w times $t_1$ to $t_2$ takes place in such a way that

$$\delta \int^{t_2} L\,dt = 0 \quad \text{--- (1)}$$

where $\boxed{L = T - V}$ is known as Lagrangian. The motion of the system take place in such a way that the variation of the integral of Lagrange w.r.t. time is zero b/w $t_1$ to $t_2$

$$\delta \int_{t_1}^{t_2} L(q_1, q_2, \ldots q_n, \dot{q}_1, \dot{q}_2 \ldots \dot{q}_n, t)\,dt = 0 \quad \text{--- (2)}$$

Proof:

Let us consider of conservation a system of particle employing the generalized coordinate int. can be written as

$$\int_{t_1}^{t_2} T(q_j, \dot{q}_j) - V(q_j)\,dt$$

According the Hamilton's principle we have

$$\delta \int_{t_1}^{t_2} T(q_j, \dot{q}_j) - V(q_j)\, dt = 0$$

$$\int_{t_1}^{t_2} \sum_j \frac{\partial T}{\partial q_j} \delta q_j + \frac{\partial T}{\partial \dot{q}_j} \delta \dot{q}_j - \frac{\partial V}{\partial q_j} \delta q_j \Big]\, dt = 0$$

$$\int_{t_1}^{t_2} \sum_j \left(\frac{\partial T}{\partial q_j} - \frac{\partial V}{\partial q_j}\right) \delta q_j + \int_{t_1}^{t_2} \sum_j \frac{\partial T}{\partial \dot{q}_j} \frac{d}{dt} (\delta q_j)\, dt = 0$$

on Integrating by part we get

$$\int_{t_1}^{t_2} \sum_j \left(\frac{\partial T}{\partial q_j} - \frac{\partial V}{\partial q_j}\right) \delta q_j\, dt + \left[\sum_j \frac{\partial T}{\partial \dot{q}_j} \delta q_j\right]_{t_1}^{t_2}$$

$$- \int_{t_1}^{t_2} \sum_j \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) \delta q_j\, dt = 0$$

Since in a such a variation, there is no Coordinate variation at end points $(\delta q_j)_{t_1}^{t_2} = 0$ so hence the Equation

reduce to

$$\int_{t_1}^{t_2} \sum_j \left(\frac{\partial T}{\partial q_j} - \frac{\partial V}{\partial q_j}\right) \delta q_j\, dt \Big] - \int_{t_1}^{t_2} \sum_j \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) \delta q_j\, dt =$$

$$\int_{t_1}^{t_2} \sum_j \left(\frac{\partial T}{\partial q_j} - \frac{\partial V}{\partial q_j} - \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right)\right] \delta q_j\, dt = 0$$

Since Each $\delta q_j$ are independent of Each other the coefficients of Every $\delta q_j$ should be zero to satisfy the above Eq (the coefficients of $j^{th}$ coordinate zero), we get


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$$\left[ \frac{\partial T}{\partial q_j} - \frac{\partial V}{\partial q_j} - \frac{d}{dt}\left( \frac{\partial T}{\partial \dot{q}_j} \right) \right] = 0$$

$$\frac{d}{dt}\left( \frac{\partial T}{\partial \dot{q}_j} \right) - \frac{\partial}{\partial q_j}\left( T - V \right) = 0$$

- Now for conservation system V is not a function of velocities of $q_j$ but only of coordinate then force

$$\frac{d}{dt} \frac{\partial (T-V)}{\partial \dot{q}_j} - \frac{\partial (T-V)}{\partial q_j} = 0$$

Hence we introduced the concept of scalar function is called Lagrange (L) for a scalar function of $q_j, \dot{q}_j, t$ thus the above equation takes the form

$$\boxed{\sum_j \left[ \frac{d}{dt}\left( \frac{\partial L}{\partial \dot{q}_j} \right) - \frac{\partial L}{\partial q_j} \right] = 0}$$

This set of equation is called Lagrangian equation of motion each is IInd order differential eq in terms of the time as independent variable


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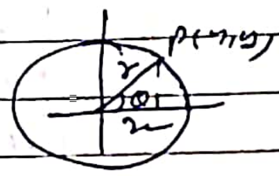
Generalised Coordinate (Space - Coordinate) :- These minimum Coordinate which is used to describe the configuration of Any particle at Any Instant is Called Generalise Coordinate.

Eg 1. When a particle is moving in circular motion then the Coordinate are Needed to describe the motion of the particle can be written as

$$x = r\cos\theta$$
$$y = r\sin\theta$$




$$r = \sqrt{x^2+y^2} = q_1$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) = q_2.$$

Here r And θ is Known as generalised Coordinate And denoted by $q_1$ & $q_2$.
from above it is clear that the Generalised Coordinate may be Cartesian (x, y, z) form, Cylindrical (r, φ, z) & Polar form (r, θ, φ).

Eg 2. When a particle is moving in spherical then a Relation b/w Cartesian Coordinate And polar Coordinate can be written as

$$x = r\sin\theta\cos\phi$$
$$y = r\sin\theta\sin\phi$$
$$z = r\cos\theta$$

$$r = \sqrt{x^2+y^2+z^2} = q_1$$

$$\theta = \tan^{-1}\frac{\sqrt{x^2+y^2}}{z^2} = q_2$$

$$\phi = \tan^{-1}\left(\frac{y}{x}\right) = q_3$$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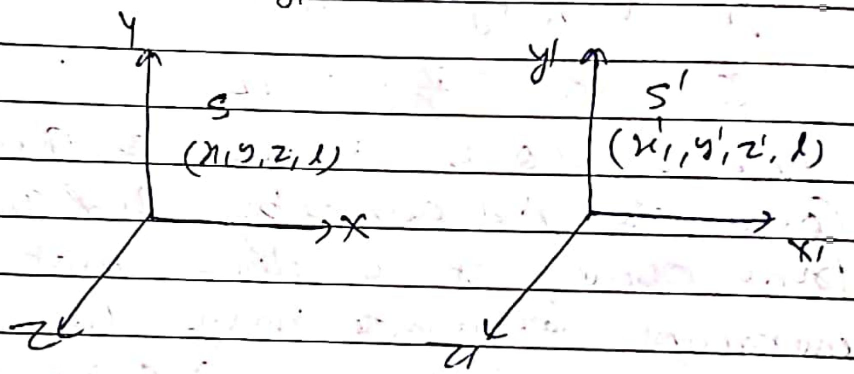
here $r, \theta, \phi$ is known as generalised Coordinate & denoted by $q_1 q_2 \cdot q_3$
from above it is clear that to discuss the Configuration of Any System maximum three Coordinates are needed.

Eq. 3. In the Relativity when a frame of Reference $S$ is Constant while a moving with Velocity $v$ w.r.to $S$ frame of Refrance then the Relative motion b/w these frame of Refrance is given as




$$x' = \frac{x - ut}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$y' = y \quad z' = z$$

Properties of Generalised Coordinate

1. to describe the Configuration of Any System max three generalised Coordinate are possible.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2. Generalised Coordinate may be Cartesian form. polar form, cylindrical form.

3. It is used to calculate the degree of freedom of any particle in any system i.e.

$$\boxed{\text{Degree of freedom} = 3N - K}$$

It means three Coordinate are possible to calculate the D.O.F.

4. It does not change the Constraint Condition but the Constraint Condition deduce the value of generalised Coordinate.


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