$[H, \ldots]_{q,p} = 0 - [H, \ldots]_{q,p}$

* Cyclic Coordinate or Ignorable Coordinate :-

Those Coordinate which does not Appear in the Lagrangian of Any System is Known as Cyclic Coordinate or Ignorable Coordinate.

Let us Consider a Coordinate q_k which does Not Appear in the Lagrange of Any System then, q_k is said to be cyclic Coordinate & in Mathematically form Can be Expressed as.

$$\frac{\partial L}{\partial q_k} = 0$$

Let us Consider a Lagrangian which Can be Expressed in the form of

$$L = \frac{1}{2} m(\dot{r}^2 + r^2 \dot{\theta}^2) + \frac{K}{r}$$

which is the Equation of Centre of force then in above Lagrangian the Coordinate Q is Not present then Q is said to be cyclic Coordinate & can be Expressed in the form of

$$\boxed{\frac{\partial L}{\partial Q} = 0}$$

And we know that cyclic Coordinate can be defined as

$$\boxed{\frac{\partial L}{\partial q_k} = 0}$$

then

$$\boxed{q_k = 0}$$

* Show that the Transformation

$$Q = \log(1+\sqrt{q}\cos p)$$
$$P = 2\sqrt{q}\,[1+\sqrt{q}\cos p]\sin p \quad \text{is}$$

Canonical And find the Generating function $F[p, P]$.

Soln:- $[P, Q]_{q,p}$

$$\frac{\partial P}{\partial q}\frac{\partial Q}{\partial p} - \frac{\partial P}{\partial p}\frac{\partial Q}{\partial q}$$

$$p\,dq - P\,dQ \quad \text{—— (1)}$$

we have

$$Q = \log(1+\sqrt{q}\cos p)$$

$$dQ = \frac{1}{1+\sqrt{q}\cos p}\left[\cos p \times \frac{1}{2\sqrt{q}} + \sqrt{q}\sin p\right]$$

$$dQ = \frac{1}{1+\sqrt{q}\cos p}\left[\frac{1}{2\sqrt{q}}\cos p - \sqrt{q}\sin p\right]$$

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$$dQ = \frac{1}{(1+\sqrt{q}\cos p)}\left[\frac{\cos p - 2q\sin p}{2\sqrt{q}}\right]$$

$$dQ = \frac{\cos p - 2q\sin p}{2\sqrt{q} + 2q\cos p}$$

$$dQ = \frac{\cos p - 2q\sin p}{2\sqrt{q}[1+\sqrt{q}\cos p]}$$

$$dQ = \frac{\cos p\,dq - 2q\sin p\,dp}{2\sqrt{q}(1+\sqrt{q}\cos p)}$$

Substitute the value of P & dQ in Eq (1)

$$pdq - \cancel{2\sqrt{q}}\cancel{[1+\sqrt{q}\cos p]}\sin p \times \frac{(\cos p\,dq - 2q\sin p\,dp)}{\cancel{2\sqrt{q}}\cancel{(1+\sqrt{q}\cos p)}}$$

$$\cancel{pdq - \sin p\,dq + 2q\sin^2 p\,dp}$$

$$pdq - \sin p\cos p\,dq + 2q\sin^2 p\,dp$$

$$pdq - \frac{1}{2}\times 2\sin p\cos p\,dq + 2q\sin^2 p\,dp$$

$$pdq - \frac{1}{2}\sin 2p\,dq + 2q\sin^2 p\,dp$$

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$$(p - \tfrac{1}{2}\sin 2p)dq + 2q\sin^2 p\,dp + q(1-\cos 2p)\,dp$$

$$= d\left\{q\left(p - \tfrac{1}{2}\sin 2p\right)\right\}$$

Which is an exact diff. Eq. Hence the given transformation is Canonical

Now we have

$$F(p, Q, t)$$

$$Q = \log(1+\sqrt{q}\cos p)$$

$$q = (e^Q - 1)^2 \sec^2 p \qquad \left\{\sec x = \frac{1}{\cos x}\right\}$$

$$q = \left(\frac{e^Q - 1}{\cos p}\right)^2 \text{Ans}$$

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We know that

$$q = -\frac{\partial F}{\partial p}$$

$$p = -\frac{\partial F}{\partial q}$$

So $q = -\frac{\partial F}{\partial p} = (e^q-1)^2 \sec^2 p$

$$\frac{\partial F}{\partial p} = -\sec^2 p\,(e^q-1)^2$$

$$dF = -(e^q-1)^2 \sec^2 p\, dp$$

Taking Integration

$$\int dF = -(e^q-1)^2 \int \sec^2 p\, dp$$

$$\boxed{F = -(e^q-1)^2 \tan p} \text{ Proved}$$

for 5 marks.

Q1 Discuss Stable And Unstable Equilibrium for a System Excluding Small Oscillation also. Q Obtain Lagrang Eq of motion for Small oscillation

QN2. A cylinder is rolling down an Inclined plane Calculate its velocity at the bottom of the Inclined plane

QN3. Show that the Transformation

$$Q = \log(1+\sqrt{q}\cos p)$$

$P = 2\sqrt{q}\,(1+\sqrt{q}\cos p)\sin p$ find Generating function $F(P, Q)$

Q Write the Generalised Coordinate.

(i) Simple Pendulum (ii) Fly wheel. (iii) particle on the Surface of sphere (iv) beads of An Abacus

2 marks

Q State And Prove Kepler Second Law of planetary motion.

2 marks

Q. State and prove virial theorem

Q. Explain Euler theorem and also explain Euler equation of motion

Q. What is small oscillation also define stable and unstable equilibrium

Q.

10 marks What do you understand by generalised coordinate. Obtain the d-Alembert principle in generalised coordinate and use it to obtain Lagrange eq of motion for a holonomic conservative system. (L.S. page)

(2.) What do you understand by cyclic coordinate (ignorable coord). Prove that if a coordinate is not present in the Lagrange then it will not present in Hamiltonian

(3) What are the Kepler law of planetary motion. Prove them and hence show that areal velocity is constant.