Imp Q. 2 marks

Q. Show that $F = -\sum_i q_i P_i$ represent space inversion

Ans. Since generating F is a function of old coordinate & new momentum, i.e. of second type, hence from the transformation eq we know that

$$p_i = \frac{\partial F}{\partial q_i} = -\frac{\partial}{\partial q_i} \sum_i q_i P_i$$

$$= -P_i$$

$$Q_j = \frac{\partial F}{\partial P_i} = -\frac{\partial}{\partial P_i} \sum_i q_i P_i = -q_j$$

$$\& \; K = H + \frac{\partial F}{\partial t} \quad [K \,\&\, H \text{ are Hamiltonian function}]$$

$$= H$$

Since $F \neq F(t)$, the function F generate the identity transformation. This shows the fact that space inversion constitute of a special case of a canonical transformation

Q. State and Prove Virial theorem

Q. What is small oscillation. Explain Normal coordinate also define stable & unstable equilibrium

Q. State and Prove Kepler IInd law

Q. Find out Lagrangian eq for a L-C circuit

Ans. The lagrangian for an electrical circuits

$$L_E = T_m - V_E$$

where T_m = Magnetic Energy

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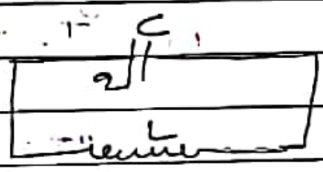
Lets Consider an Electric circuit Containing Inductance L And Capacitance C.



V_E = Electrical Energy

$$T_m = \frac{1}{2} L i^2 = \frac{1}{2} L \dot{q}^2$$

$$V_E = \frac{1}{2}\frac{q^2}{c}$$

$$L_E = \frac{1}{2} L \dot{q}^2 - \frac{1}{2}\frac{q^2}{c} \quad (1)$$

Choosing q as generalize Co-ordinate Eq of motion

$$\frac{d}{dt}\left(\frac{\partial L_E}{\partial \dot{q}}\right) - \frac{\partial L_E}{\partial q} = 0$$

$$\frac{\partial L_E}{\partial \dot{q}} = L\dot{q} \qquad \frac{\partial L_E}{\partial q} = -\frac{q}{c}$$

$$\boxed{L\ddot{q} + \frac{q}{c} = 0}$$

Q. Write the Generalise Coordinate for the System

(1) Simple pendulum - the Angle which the Pendulum make with the Vertical Line through point of Suspension.

(2) particle on the surface of sphere - (θ, ϕ), as the usual polar Angle of a point on the sphere.

(3) Fly wheel - (θ) - the Angle b/w a definite radius of the fly wheel And fixed line Perpendicular to the Axis

(4) Beads of abacus - x_i the Cartesian Coordinate along the horizontal wire.

Q. Explain Jacobi form of the Least Action principle -

Transformation Eq do Not Involve time so K.E can also be Expressed as homogeneous quadratic function of

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Velocity

i.e. $T = \frac{1}{2} \sum_{jk} a_{jk} \dot{q}_j \dot{q}_k$

$$2T = \sum_{jk} a_{jk} \dot{q}_j \dot{q}_k$$

$$2T = \sum_{jk} a_{jk} \frac{dq_j \, dq_k}{(dt)^2}$$

$$2T = \left(\frac{d\rho}{dt}\right)^2 \quad \text{--- (1)} \Rightarrow dt^2 = \frac{(d\rho)^2}{2T}, \text{ Taking under root}$$

$$dt = \frac{d\rho}{\sqrt{2T}}$$

where $(d\rho)^2 = \sum_{jk} a_{jk} dq_j \, dq_k$

Now Principal of Least Action after putting the values of dt from Eq (1)

$$\Delta \int_{t_1}^{t_2} 2T \, dt = \Delta \int_{t_1}^{t_2} 2T \frac{1}{\sqrt{2T}} d\rho$$

$$= \Delta \int_{t_1}^{t_2} \sqrt{2T} \, d\rho = 0$$

For such a System we can write

$H = T + V(q)$

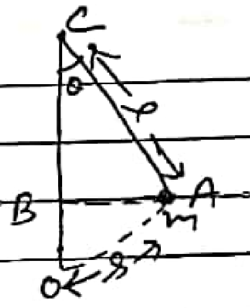
$T = H - V(q)$

then $\Delta \int_{t_1}^{t_2} \sqrt{2(H - V(q))} \, d\rho = 0$

Which is the Term as Jacobi form Principal of Least Action

(Extra)

Let θ is the Angular displacement from the Equilibrium position. l is the length of the Thread And m is mass of the bob then we know that



$$T = \frac{1}{2} m v^2 \quad \text{—— (1)}$$

$$\text{कोण} = \frac{\text{चाप}}{\text{त्रिज्या}}$$

$$\text{Angle} = \frac{\text{Arc}}{\text{radius}}$$

$$\theta = \frac{s}{l}$$

$$s = l\theta$$

we know that

$$v = \frac{\text{दूरी}}{\text{समय}} = \frac{ds}{dt}$$

put the value of s

$$v = \frac{d}{dt}(l\theta)$$

$$v = l\frac{d\theta}{dt}$$

$v = l\dot{\theta}$ put in (1)

$$T = \frac{1}{2} m (l\dot{\theta})^2$$

$$\boxed{T = \frac{1}{2} m l^2 \dot{\theta}^2} \quad \text{—— (2)}$$

we also know that

$$I = ml^2$$

$$\boxed{T = \frac{1}{2} I \dot{\theta}^2}$$

or

$$T = \frac{1}{2} m v^2 \quad (1)$$

$$v = r\omega$$

$$v = l\omega$$

$$\omega = \frac{d\theta}{dt}$$

$$\omega = \dot{\theta}$$

$$v = l\dot{\theta}$$

$$T = \frac{1}{2} m (l\dot{\theta})^2$$

$$\frac{1}{2} m l^2 \dot{\theta}^2$$

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Q What are Cyclic or Ignorable Coordinate also Explain Conservation of Linear Momentum

Q. Explain Euler theorem And also Explain Euler Equation of motion for a rigid body

Ans Euler theorem states that in 3-D space the General Displacement of a rigid body with one fixed point is equivalent to a single rotational about same Axis of Produced by the Rotation Angle ie known as An Axis Angle

$$\boxed{\lambda_1 \lambda_2 = 1}$$

Euler equation of motion for a rigid body

$$N_x = I_1 \dot{w}_x - (I_2 - I_3) w_y w_z$$

$$N_y = I_2 \dot{w}_y - (I_3 - I_1) w_z w_x$$

$$N_z = I_3 \dot{w}_z - (I_1 - I_2) w_x w_y$$

5 marks

Q.1. Discuss stable And unstable Equilibrium for a system Executing small oscillation. obtain lagrangian Equation of motion for small oscillation.

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