UNIT-5

Stable And Unstable Equalibrium:-

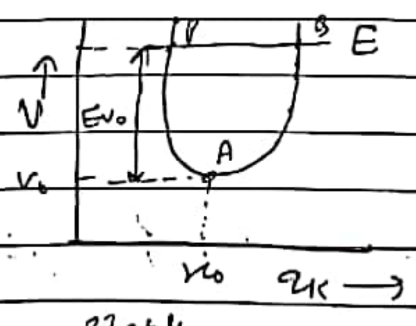
If a particle is in motion or in Rest Remain always in the same position by Applying the External force is known as Stable Equalibrium

Such time a Condition is obtained both the part which make two and fro motion about the fixed point and all Remain in Rest.

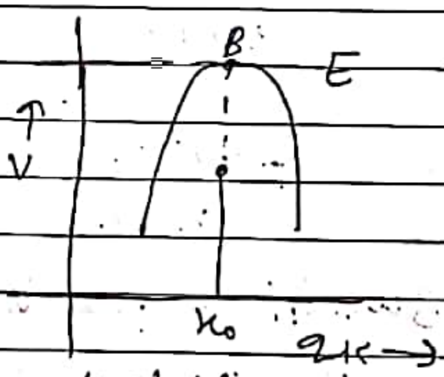
Example- bar Pendulam

If the particle does not Remain at the Stable Condition said to be Unstable Equalibrium It means that when force is Applied on the particle and it's Motion Change Continuously is known as Unstable Equalibrium

EX-A Arod Stating on it's one End,
or A Egg " " " " "

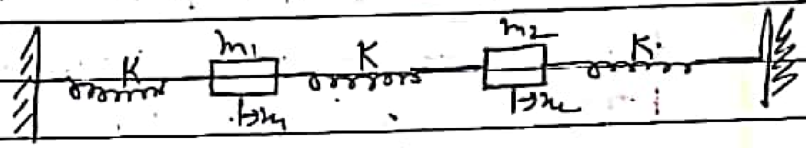


Stable



unstable

two Coupled motion or oscillator:-



$m_1 = m_2 = m$

let us Consider two mass m_1 & m_2 which are attached from two walls by string.
let x_1 is the position of first mass from its walls And x_2 is the position of Second mass from second wall And. K is spring Constant which is shown in figure
If the spring is stretched then both mass begin to Vibrate about their mean position then the K.E. Can be Expressed as

$$T = \frac{1}{2} m\dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}_2^2 \quad -(1)$$

If the masses in Rest position then the P.E. Can be written as

$$V = \frac{1}{2} K x_1^2 + \frac{1}{2} K x_2^2 + \frac{1}{2} K(x_2 - x_1)^2 \quad -(2)$$

where K is a spring Constant whose Value is

$$K = \omega_0^2 m \quad -(3)$$

Now from the definition of Lagrangian is the difference of K.E And P.E is L

i.e

$L = T - V$ [illegible]

$$L = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}_2^2 - \frac{1}{2} \omega_0^2 m \left\{ x_1^2 + x_2^2 + (x_2 - x_1)^2 \right\}$$

if the mass of the Particle is same than the above Equation Reduced in the form of ie $[m_1 = m_2 = m]$

$$L = \frac{1}{2} m (\dot{x}_1^2 + \dot{x}_2^2) - \frac{1}{2} \omega_0^2 m \left\{ x_1^2 + x_2^2 + (x_2 - x_1)^2 \right\} \quad -(4)$$

Now from the Legrangian Equation we know that

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) - \frac{\partial L}{\partial x_1} = 0 \quad -(5)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_2}\right) - \frac{\partial L}{\partial x_2} = 0 \quad -(6)$$

$$\frac{\partial L}{\partial \dot{x}_1} = m\dot{x}_1 \qquad \frac{\partial L}{\partial \dot{x}_2} = m\dot{x}_2$$

$$\frac{\partial L}{\partial x_1} = -\omega_0^2 m (2x_1 - x_2)$$

$$\frac{\partial L}{\partial x_2} = -\omega_0^2 m (2x_2 - x_1)$$

Substituting these value Eq (5) & (6) we get

$$\frac{d}{dt}(m\dot{x}_1) + \omega_0^2 m (2x_1 - x_2) = 0$$

$$m\ddot{x}_1 + 2\omega_0^2 m x_1 - \omega_0^2 m x_2 = 0$$

$$\ddot{x}_1 + 2\omega_0^2 x_1 - \omega_0^2 x_2 = 0 \quad -(7)$$

$$\frac{d}{dt}(m\dot{x}_2) + \omega_0^2 m (2x_2 - x_1) = 0$$

$$\ddot{x} + 2\omega_0^2 x_2 - \omega_0^2 x_1 = 0 \quad -(8)$$

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Equation (7) & (8) are the second order differential equation

whose soln can be expressed as

$$x_1 = Ae^{\alpha t}$$
$$x_2 = Be^{\alpha t}$$
$$\ddot{x}_1 = A\alpha^2 e^{\alpha t} \text{ And } \ddot{x}_2 = B\alpha^2 e^{\alpha t}$$

$\ddot{x}_1$ की substitute then value in Eq (7) we get

$$\alpha^2 e^{\alpha t} A + 2w_0^2 Ae^{\alpha t} - w_0^2 Be^{\alpha t} = 0$$

$$e^{\alpha t}(\alpha^2 A + 2w_0^2 A - w_0^2 B) = 0$$

Since $e^{\alpha t} \neq 0$

then bracket Term should be Equal to zero

$$\alpha^2 A + 2w_0^2 A - w_0^2 B = 0$$

$$A(\alpha^2 + 2w_0^2) - w_0^2 B = 0 \quad \text{--- (9)}$$

Now Again substitute the value of $\ddot{x}_1$ And $\ddot{x}_2$ in Eq (8) we get.

$$B\alpha^2 e^{\alpha t} + 2w_0^2 x_2 - w_0^2 x_1 = 0$$

$$e^{\alpha t}(\alpha^2 B + 2w_0^2 B - w_0^2 A) = 0$$

Since $e^{\alpha t} \neq 0$

$$\alpha^2 B + 2w_0^2 B - w_0^2 A = 0$$

$$-Aw_0^2 + B(\alpha^2 + 2w_0^2) = 0 \quad \text{--- (10)}$$

Now to calculate the value of A & B the above equation can be written in determinant form which can be expressed as

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$$\begin{vmatrix} \alpha^2 + 2\omega_0^2 & -\omega_0^2 \\ -\omega_0^2 & \alpha^2 + 2\omega_0^2 \end{vmatrix}$$

$$\alpha = \pm i\omega_0, \pm\sqrt{3}\, i\omega_0$$

Now putting these value in the soln of Equation we get

$$x_1 = Ae^{\pm i\omega_0 t}$$

$$x_2 = Be^{\pm\sqrt{3}i\omega_0 t}$$

these are the displacements of masses m_1 & m_2 from fixed point when both masses are in motion or in Rest position ie the condition of stability and unstability exist.