papar

* Hamilton Canonical Application.

(1). Simple pendulum

We know that in a Simple Pendulum the K.E of the particle can be written

$$T = \frac{1}{2} m v^2$$

$$T = \frac{1}{2} m l^2 \dot{\theta}^2 \quad \text{--- (1)}$$

$v = r\omega$, $v = l\omega$, $v = l\dot{\theta}$, $T = \frac{1}{2} m v^2 = \frac{1}{2} m l^2 \dot{\theta}^2$

P.E can be written as

$$V = mgh$$

$$V = mgl(1 - \cos\theta) \quad \text{--- (2)}$$

Now by the definition of lagrange is the diff of K.E and P.E i.e

$$L = T - V$$

$$L = \frac{1}{2} m l^2 \dot{\theta}^2 - \{mgl(1 - \cos\theta)\} \quad \text{(3)}$$

~~partialy diff w.r.to $\dot{\theta}$ we get~~

$$p = \frac{\partial L}{\partial \dot{\theta}} = m l^2 \dot{\theta}$$

$$p = m l^2 \dot{\theta}$$

$$\dot{\theta} = \frac{p}{m l^2} \quad \text{--- (4)}$$

~~Now by the definition of Hamiltonian is the sum of P.E and K.E~~

$$H = T + V$$

$$H = \frac{1}{2} m l^2 \dot{\theta}^2 + mgl(1 - \cos\theta)$$

Substitute the value of $\dot{\theta}$ we get

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[redacted]

$$H = \frac{1}{2} m l^2 \left(\frac{p}{ml^2}\right)^2 + mgl(1-\cos\theta)$$

$$H = \frac{p^2}{2ml^2} + mgl(1-\cos\theta) \quad \text{--- (5)}$$

Now from Hamilton Canonical Eq we have

$$\dot{p} = -\frac{\partial H}{\partial \theta}$$

$$\dot{\theta} = \frac{\partial H}{\partial p}$$

Now diff w. r to p and θ of Eq (5) we get

$$\frac{\partial H}{\partial p} = \frac{2p}{2ml^2} = \frac{p}{ml^2} = \dot{\theta}$$

$$\frac{\partial H}{\partial \theta} = -mgl \sin\theta = -\dot{p}$$

Now from Eq (4)

$$\dot{\theta} = \frac{p}{ml^2}$$

diff w. r to t

$$\ddot{\theta} = \frac{\dot{p}}{ml^2}$$

$$\ddot{\theta} = -\frac{mgl \sin\theta}{ml \times l}$$

$$\ddot{\theta} = -\frac{g \sin\theta}{l}$$

Since θ is very small

$$\ddot{\theta} = -\frac{g\theta}{l} \quad \{\sin\theta = \theta\}$$

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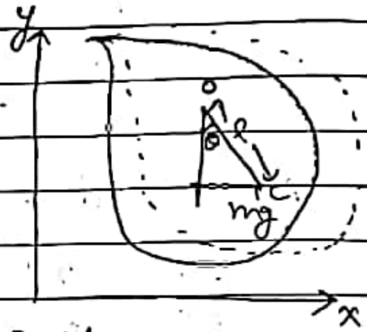
$$\boxed{\ddot{\theta} + \frac{g}{l}\theta = 0}$$

This is the required Eq for simple pendulam.

M. Imp.

* Compound Pendulam :-



The lagrangian for a compound pendulam is obtained from T

$$T = \frac{1}{2} I \dot{\theta}^2$$

$$V = -mgl \cos\theta$$

Now from the definition of lagrangian is the difference of K.E and P.E ie

$$L = T - V$$

$$L = \frac{1}{2} I \dot{\theta}^2 + mgl \cos\theta$$

differentiate w.r.to $\dot{\theta}$ we have

$$\frac{\partial L}{\partial \dot{\theta}} = I\dot{\theta} = P$$

$$\dot{\theta} = \frac{P}{I}$$

Now from Hamiltonian is the sum of P.E and K.E ie

$$H = T + V$$

$$H = \frac{1}{2} I \dot{\theta}^2 - mgl \cos\theta$$

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Substituting the value of $\dot{\theta}$ we get

$$H = \frac{1}{2} I \dot{\theta}^2 - mgl\cos\theta$$

$$H = \frac{1}{2} I \frac{P^2}{I^2} - mgl\cos\theta$$

$$H = \frac{1}{2} \frac{P^2}{I} - mgl\cos\theta$$

diff w r to P and θ

$$\frac{\partial H}{\partial P} = \frac{P}{I}$$

$$\frac{\partial H}{\partial \theta} = mgl\sin\theta$$

Now from the Hamilton canonical Eq we have

$$\dot{\theta} = \frac{\partial H}{\partial P}, \quad -\frac{\partial H}{\partial \theta} = \dot{P}$$

$$\text{So} \quad \frac{\partial H}{\partial P} = \frac{P}{I} = \dot{\theta} \longrightarrow \left[\begin{matrix} P = I\dot{\theta} \\ \dot{P} = I\ddot{\theta} \end{matrix} \right]$$

$$\text{And} \quad \dot{P} = -\frac{\partial H}{\partial \theta} = -mgl\sin\theta$$

$$I\ddot{\theta} = -mgl\sin\theta$$

Since θ is very small

so $\sin\theta \approx \theta$

$$I\ddot{\theta} = -mgl\theta$$

$$mgl\theta + I\ddot{\theta} = 0$$

$$\boxed{\ddot{\theta} + \frac{mgl}{I}\theta = 0}$$

Which is the equation of motion of compound pendulum with time period.

$$\boxed{\omega = \sqrt{\frac{mgl}{I}}}$$

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