$$\text{द्रव्यमान प्रतिशत (\%)} = \frac{\text{विलय पदार्थ का द्रव्यमान}}{\text{विलयन का कुल द्रव्यमान}}$$

* आंकिक प्रश्न :-

Q.1. यदि 22 ग्राम बेन्जीन मे 122 ग्राम कार्बन टेट्रा क्लोराइड घुली हो तो बेन्जीन एवं कार्बन टेट्रा क्लोराइड के द्रव्यमान प्रतिशत क्या होंगे ?

* Classical Mechanics :- *

Constraints :- The Restriction or boundation which is Applied on a particle to Restrict it's motion is called Constraints.

or

the Restriction which is Applied on a particle to restrict the motion of a particle of a particular path is called Constraints.

Case-I. for a circle motion, motion on the circumference Eq of motion is

$x^2+y^2=r^2$

Case-II- for a sphere. Eq of motion is $x^2+y^2+z^2=r^2$

Types of Constraints :-

1- Holonomic Constraints :- The Constraints Expressed in the form of

$f(x_1, y_1, z_1, x_2, y_2, z_2 - - t) = 0$

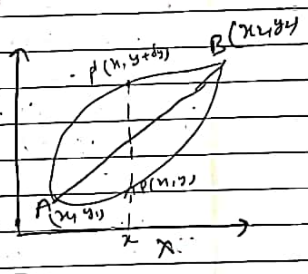
is called Holonomic Constraints

Ex. Rigid body.

Let us Consider a function $F(y, \dot{y}, x)$ defined on a curve given by $y = y(x)$. b/w the two points $A(x_1, y_1)$ and $B(x_2, y_2)$ here $\dot{y} = dy/dx$. We want to find the particular curve $y(x)$ for which the line Integral I is the function of F b/w the two points.

* Euler Lagrangian Eq :-

In this figure there are two points A, B whose Generalized Co-ordinate are given that $A(x_1, y_1)$, $B(x_2, y_2)$



Now we know that

$$I = \int F(y, \dot{y}, x)\, dx \quad \text{—} (1)$$

Now Applying the Variation integral principle

$$\delta I = \delta \int F(y, \dot{y}, x)\, dx = 0 \quad \text{—} (2)$$

Now Let us Consider parameter which gives shortest distance b/w two points A & B

So α is the parameter when is the function of y and $\dot{y}$ and Not the function of x

Then the Equation Ist may be written as.

$$I = \int F(y, \alpha), (\dot{y}, \alpha), x \,]\, dx \quad \text{—} (3)$$

differentiate w.r.to α we get:

$$\frac{dI}{d\alpha} = \int \left[\frac{\partial F}{\partial y} \frac{\delta y}{d\alpha} + \frac{\delta F}{\partial \dot{y}} \frac{\delta \dot{y}}{d\alpha} + \frac{\partial F}{\partial x} \frac{dx}{d\alpha} \right] dx$$

Since x is Not function of α then the Third Term of the above integral $= 0$

$$\frac{dI}{d\alpha} = \int \left(\frac{\partial F}{\partial y} \frac{dy}{d\alpha} + \frac{\partial F}{\partial \dot{y}} \frac{d\dot{y}}{d\alpha} \right) d\alpha$$

$$\frac{dI}{d\alpha} = \left[\int_{x_1}^{x_2} \frac{\partial F}{\partial y} \frac{dy}{d\alpha} + \int_{x_1}^{x_2} \frac{\partial F}{\partial \dot{y}} \frac{d\dot{y}}{d\alpha} \right] dx$$

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Now the Integration of the IInd Term of the above Eq we get

$$\frac{dI}{d\alpha} = \left[\int_{x_1}^{x_2}\left(\frac{\partial f}{\partial y}\frac{dy}{d\alpha}\right)dx + \left(\frac{\partial f}{\partial y}\frac{dy}{d\alpha}\right)_{x_1}^{x_2} - \int_{x_1}^{x_2}\frac{d}{dx}\left(\frac{\partial f}{\partial y}\right)\frac{dy}{d\alpha}dx\right]$$

Since this Equation that is in the form x_1 to x_2. then IInd Term of above Eq is Zero

$$\frac{dI}{d\alpha} = \int_{x_1}^{x_2}\left(\frac{\partial f}{\partial y}\frac{dy}{d\alpha}\right)dx - \int_{x_1}^{x_2}\left[\frac{d}{dx}\left(\frac{\partial f}{\partial y}\right)\frac{dy}{d\alpha}\right]dx$$

$$\frac{dI}{d\alpha} = \int_{x_1}^{x_2}\left[\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y}\right)\right]\frac{dy}{\partial\alpha}.dx$$

Now from the Variation Principle we know that

$$\frac{\delta I}{d\alpha} = 0$$

$$\int_{x_1}^{x_2}\left[\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y}\right)\right]\frac{dy}{d\alpha}.dx = 0$$

$$\int_{x_1}^{x_2}\frac{dy}{\partial\alpha}dx \neq 0$$

Now the bracket Term should be equal to Zero

$$\boxed{\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial \dot{y}}\right) = 0}$$

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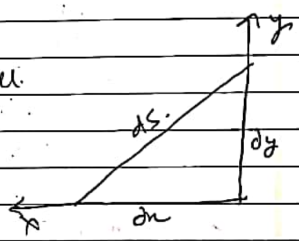
This is Euler Lagrangian Eq which is dependent on the position of particle and it's derivative and also parameter of position coordinate.

* Application or Cases of Euler Lagrangian Eq.

Application or Case- I

(1) Shortest distance b/w two points in xy plane:-

Let us consider a plane in 2D in which the small components are dx, dy. Since the shortest distance b/w two points then we know that



$$ds^2 = dx^2 + dy^2$$

$$ds^2 = dx^2\left[1+\left(\frac{dy}{dx}\right)^2\right]$$

$$\frac{ds^2}{dx^2} = 1+\left(\frac{dy}{dx}\right)^2$$

$$\frac{ds^2}{dx^2} = 1+\dot{y}^2$$

$$ds = \sqrt{1+\dot{y}^2} \times dx$$

Integrate on both side we get

$$I = \int ds = \int \sqrt{1+\dot{y}^2}\,.\,dx \quad \text{--- (1)}$$

We know that on integrating the function form can be written as

$$I = \int F dx \quad \text{--- (2)}$$

from (i) & (ii)

$$F = \sqrt{1+\dot{y}^2}\,dx \quad \text{—③}$$

diff w.r.to $\dot{y}$

$$\frac{\partial f}{\partial \dot{y}} = \frac{\dot{y}}{\sqrt{1+\dot{y}^2}}$$

by Euler Lagrangian Eq.

$$\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial \dot{y}}\right) = 0$$

$$\therefore \frac{\partial f}{\partial y} = 0$$

$$-\frac{d}{dx}\left(\frac{\dot{y}}{\sqrt{1+\dot{y}^2}}\right) = 0$$

$$\frac{d}{dx}\left(\frac{\dot{y}}{\sqrt{1+\dot{y}^2}}\right) = 0$$

ON Integrate we get

$$\frac{\dot{y}}{\sqrt{1+\dot{y}^2}} = a \text{ (constant)}$$

$$\dot{y} = a\sqrt{1+\dot{y}^2}$$

square on both side

$$\dot{y}^2 = a^2\left(\sqrt{1+\dot{y}^2}\right)^2$$

$$\dot{y}^2 = a^2(1+\dot{y}^2)$$

$$\dot{y}^2 = \frac{a^2}{\sqrt{1-a^2}} = c^2 \text{ (constant)}$$

$$\dot{y}^2 = c^2$$

$$\dot{y} = c$$

$$\frac{dy}{dx} = c$$

$$dy = c\,dx$$

ON Integrate we get

$$\boxed{y = cx + d}$$

this is the Equation of straight line then we can say that shortest distance b/w two points always the straight line

\# 2. The Surface Area of Revolution is minimum:-
let us Consider a particle which is moving in x-direction in xy plane then the maximum Area of this particle can be written as.

$$dA = 2\pi x ds \quad \text{--- (1)}$$

ds = shortest distance of the plane
we know that shortest Distance b/w two points.

$$ds = \sqrt{1+\dot{y}^2}\, dx$$

$$dA = 2\pi x \sqrt{1+\dot{y}^2}\, dx$$

On Integrating

$$I = \int dA = \int 2\pi x \sqrt{1+\dot{y}^2}\, dx \quad \text{--- (2)}$$

Also Integrating in the function at form

$$I = \int F dx \quad \text{--- (3)}$$

from (2) And (3)

$$F = 2\pi x \sqrt{1+\dot{y}^2}$$

diff w.r.to y And $\dot{y}$ we get

$$\frac{\partial F}{\partial y} = 0, \quad \frac{\partial F}{\partial \dot{y}} = \frac{2\pi x \dot{y}}{\sqrt{1+\dot{y}^2}}$$

from Euler Lagrangian theorem

$$\frac{\partial F}{\partial y} - \frac{d}{dx}\left(\frac{\partial F}{\partial \dot{y}}\right) = 0$$

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$$\frac{d}{dx}\left[\frac{2\pi x y'}{\sqrt{1+y'^2}}\right] = 0$$

समाकलन अवकल को समाप्त करता है

$$\int \frac{d}{dx}(F) = F$$

on Integrating we:

$$\frac{2\pi x y'}{\sqrt{1+y'^2}} = a$$

$$2\pi x y' = a\sqrt{1+y'^2}$$

Square on both side

$$4\pi^2 x^2 y'^2 = a^2(1+y'^2)$$

$$y' = \frac{a}{2\pi\sqrt{x^2 - \left(\frac{a}{2\pi}\right)^2}}$$

on Integrating we get

$$y = \frac{a}{2\pi}\cosh^{-1}\left(\frac{x}{a/2\pi}\right) + b$$

$$\frac{2\pi}{a}(y-b) = \cosh^{-1}\left(\frac{x}{a/2\pi}\right)$$

$$\boxed{x = a\cosh\left(\frac{y-b}{a}\right)}$$

this is the equation of Surface Area Revolution:-

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